A Study of Suburbanization Effects on Urban Spatial Structure

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Chapter 1

1. Introduction

Through establishing two duo-centric models, this paper attempts to analyze the effects of the process of suburbanization on urban spatial structure and the behavior of each economic body in the city.

1-1 Trend of suburbanization

With the improvement of transportation system and the progress in technology of telecommunication, cities built after the Second World War have begun to enter their developing phase in 60's. Almost cities in the world tended suburbanization as their form of development. Castells [1989] examined employment growth in 50 largest and 50 small and mid-sized SMSAs and found that employment in office-dominated industry presents a strong increasing trend in suburbs rather than in central city. Conclusions also showed that decentralized trend were accelerating through the 1980s. Gordon & Richarson [1996] used the data of economic censuses in manufacturing, wholesaling, retail, and service industries in 1982-1987 period also to demonstrate that twelve consolidated metropolitan statistical areas (CMSAs) showed a notable decentralization trend in 80's. Beesley [1973] took a survey on a crucial English city: Leeds, and found that only during twelve years, rural land decreased from 64% in 1949 to 30% in 1961, net residential density increased in different degree in each of 49 subareas. Allison [1979] and Mills & Song [1979] respectively demonstrated that there existed a process of suburbanization in Japan and South Korea during the middle half of twentieth century.

With the emergency of the process of suburbanization, the conventional monocentric model of urban employment location is largely irrelevant to modern cities. As Wheaton [1979] appealed at the beginning of 80's, that a large and increasing share of urban employment is concentrated in suburban center, and urban economists in the field should move their eyes from traditional monocentric city model to the research of this new phenomenon emerged in cities.

1-1-2 Effects of improvement of transportation system on the process of suburbanization

As an objective condition of promoting the process of suburbanization, improvement of transportation system played an important role. To a great extent, the improvement of urban transportation system is basis on government investment. In the first stage of this process, expanded transport facilities are seen to reduce the effective price of travel. In the short run, this increases the number of trips and encourages greater lengths of trips. While in the long run, it encourages urban decentralization.

As for the problem of how to evaluate the effects of improvement of transportation system on urban sprawl, Rothenberg [1965] states, that the benefits of urban renewal must be measured by the aggregate change in land rents. Lind [1973] suggests that only rent changes in the immediate area are relevant for determining willingness pay. Pines and Weiss [1976] find that a weighted difference between rent increases in the affected area and rent decreases in other areas is a more appropriate measure. In the series researches of Solow and Vickrey [1971, 1973], they explore the question of what optimal amount of land should be devoted to urban transportation. Increasing such land (investment) reduces congestion and spatially stimulates the demand for residential land consumption. On the cost side, however, greater land devoted to transportation restricts the supply available for residential use. Wheaton [1977] argued that the appropriate measure of user benefits is equivalent to a general equilibrium "income compensation" value for highway investment. He suggested that the changes in land rents and urban housing that follow highway investment need not be separately considered if the forecast of highway user demand implicitly incorporates such changes. In his model, the price of travel is treated as exogenous and aggregate rental payments are included as part of consumers' income. Within his framework, transportation investment reduces the price of travel, moreover, increases welfare level.

In the analysis of traditional Alonso [1964]'s monocentric city with a transportation system of including an infinite rays coming from CBD (Central Business District), the effects of transportation costs, as a major index, is usually evaluated. Wheaton [1974] first performed comparative static analysis on traditional Alonso's model and found that improvement of transportation system expands urban size. David Pines & Efraim Sadka [1986], Sasaki [1987a] used the different way to perform comparative static analysis on Alonso's model and get the same effects of transportation cost. Brueckner [1987] also carried out the same work and obtained the same results. This conclusion provides an important theoretical evidence for urban sprawl and development, meanwhile hints likelihood existed, which improvement of urban transportation system causes emergency of the process of decentralization. As well as Castells [1989, P157] expounded that massive suburban decentralization is closely associated with the expansion of the metropolitan freeway system and the use of the automobile as the prevalent mode of commuting.

All analyzed results of the models cited above reply on a strict assumption: point CBD. Mills [1967] first relaxed the assumption, in which goods production is performed in an area CBD, while housing production is carried out in suburban area surrounding the CBD, and transportation service is produced in both areas. Each kind of production activities requires land, labor and capital input. The model is focused on how the factor inputs are allocated among three production activities in competitive markets. His model is more general than traditional models in that the demand for land in the production of transportation service as well as goods is explicitly treated. But the model is too intractable to obtain the explicit solution for the endogenous variables even specify the production function in a concrete form, i.e., the Cobb-Douglas type.

Sullivan [1983] develops general model [see also Arnott & Mackinnon, 1977, Henderson, 1975], in which lands in the city are utilized in four sectors: manufacturing's input, households' consumption, inputs in housing production and transportation sector. In his model, wage rate at each industrial location, both borders of CBD and city are endogenously determined.

Witchard [1984] performed a normative analysis, where the criterion of social optimization is to maximize the average utility of households, i.e., the average consumption of factory goods. The model assumes that factories are located in central area of a circular city, and a firm controls the amount of labor and the size of land as inputs into production. In his analysis, it is shown that as the transportation cost is lowered, the utility level and 'shadow income' level are increased, CBD area is expanded. However, city size is not necessarily enlarged since residential land consumption is independent of his location. As same as Mills[1967]'s work, the model can not parallel to the results of conventional residential location analysis, since land size for residents is treated exogenously.

Straszheim [1984] evolved an open model in which profit of a firm is assumed zero because of constant returns to scale of production and perfect competition. It makes the model general in that the location of the workplace of each resident and his wage level are endogenously determined. However, explicit solution can not be obtained because of the complexity of the system, and hence the sensitivity analysis is carried out numerically.

In Sasaki & Kaiyama [1989] an aggregative urban model, CBD is treated as an area by assuming that firms employ both labor and land to perform production in a specified linearly homogenous production function. The model is crucially focused on how the improvement of transportation system influences urban spatial structure, in which the effects of two index of freight and passenger transport cost are evaluated. In their analysis results, it is found that improvements of both freight and passenger's transportation systems tend to expand city size, although in some cases, industrial area in central area contracts. As another contribution of their work, they also demonstrate that, Wheaton [1977]'s analysis method that urban transport demand is necessary for evaluating social welfare change is not only adaptable in a pure residential location problem, but in the location problem of both producers and residents.

Improvement of urban transportation system may promote suburbs' emergency, the competition among the urban multi-transportation modes also causes the likelihood of the emergency of subcenters. Along the works of Wheaton [1977] and Sasaki & Kaiyama [1989], Sasaki [1990a] analyzed how the competition between two different transportation modes affects urban spatial structure, where transportation system change is evaluated from the viewpoints of users' welfare on the basis of the compensating variant (CV) concept. In his analytic results, it is shown that decreases in transportation cost in mode 1 (central area) reduces consumers' welfare and leads to contraction of the city, which corresponds to the trend of improving urban transportation system from mode-for-center to the mode-for-suburb. On the other hand, a reduction of commuting cost of the mode-for-suburb results in expansion of the city and increase of the rent in the periphery. In his further research Sasaki[1990b], he classifies consumers into two groups in terms of the income and obtains four urban spatial structures by combining the different income level and different transportation mode. Through performing comparative static analysis, it is obtained that improvement of transportation system in suburb expands city size in each spatial pattern. The research regarding to such two transportation modes is originally studied by Anas and Moses [1979]. They designed a model in which urban transportation system includes not only an infinite number of radial lines coming out from CBD, but a dense circular roads running around the CBD. For commuting to the center, passengers first gain access to the radial system by traveling on a dense network of streets. The dense network represents bus lines or auto travel on secondary roads while the corridors represent expressways or mass transit lines. Under these assumptions of bimodal travel, equilibrium analysis yields various urban spatial configurations. In a sense, the model is more general or realistic than basic Alonso model, but complex transport system makes comparative static analysis be intractable.

1-1-3 Effects of progress in information system on the process of suburbanization

As another objective condition of promoting the process of suburbanization, the progress in information technology also plays important role, especially, in 70's. As Toffler [1980] predicted at the beginning of 80's, that the higher technology of telecommunication would deeply influence our social configuration, life configuration and location decision making in the future. Castells [1987, chap 3] also concluded that progress in information systems and communications permit decentralization of office activities.

In promoting the process of suburbanization, the advantages of progress in information system are expressed in the following aspects:

To firms:

1. Functional linkage of decentralized units and their parent company or their market makes their pursuing low land price be possible.
2. For back office, new communication technologies make them relocate in remote locations, non-metropolitan areas or rural localities, even foreign countries.

To consumers:

1. Using computers connected to information systems networks, some professionals whose job flexibility allows them to perform work from homes.
2. For some old or weakened people, computer linkages make their shopping at home be possible. They can submit shopping list to store through networks and enjoy the service of goods-delivery.
3. For families, as long as they are on Internet, they may enjoy watching video, playing games at home through functional networks.

Indeed, progress in information systems promotes the process of decentralization; but meanwhile, it also reinforces a further centralization in the higher nodal points, which is concretely expressed in information-intensive industries. For example, enterprises of stock exchange will expand their scale or enhance their capacity of telecommunication to adapt the expanding market areas. For some non-information-intensive industries, although divisional branch offices decentralized, headquarters of corporations continue, by and large, to be situated in the CBDs of the metropolitan areas themselves (see Armstrong [1986]). They will much more centralize in nodal point for conveniently performing face-to-face communication. Therefore, in the whole process of suburbanization, previous CBD area probably contracts, but its functions are enhanced and specialized. So we may say that, to some extent, progress in information systems causes two-fold process of simultaneous decentralization and centralization.

1-2 Phenomenon of suburbanization

In the long process of the decentralization, there totally emerged three kinds of form of suburbanization in different period: satellite cities, subcenters and edge cities. In the section, we expound the three phenomenon emerge in the whole process of suburbanization and explore the distinction among the three concepts.

1-2-1 Satellite cities

Satellite cities, as original phenomenon of suburbanization, was first observed and submitted by Taylor [1915]. They experienced two phases, the first one emerged after the First World War, as well as observed by Taylor, many industrial enterprises that concentrated in central area began to disperse to periphery of metropolis and finally resulted in the formation of satellite cities. On the first phase, satellite cities showed the following characters: 1. They are formed non-planed, i.e., they are formed only because of purely factories' moving outward in order for expanding production. 2. Industry-type satellite cities dominated and industries in suburb intensely relied on central area in both of freight transportation and decision-making of management. 3. Public social facilities, such like education, entertainment were still in central area.

After the Second World War, with more and more satellite cities were built in metropolis in North America, Europe and Asia, satellite cities also entered the second phase. Comparing with the first phase, satellite cities in this phase showed the following characters: 1. Satellite cities are planning established by central government. On the other words, satellite cities in the phase play an important role of dividing metropolitan function. With the specification of metropolitan function, in order to enhance cities' management of main function, previous comprehensive metropolis transfer enterprises concerning with some insignificant functions to satellite cities. For example, China government planed to make Beijing be international political center, both heavy and light industries located in the center were gradually moved to satellite cities in suburb from 1982. 2. Living-type satellite cities dominate. In 60's, both the marriage and the birth rates continued at a high level in the world and most of countries experienced a period of baby boom (see Jackson [1985] Chap13). Over-fast growth of population brought a series serious social problems to metropolis, such as congestion, pollution, inadequate living space and so on. So satellite cities, as an efficient way for solving problems, were introduced. After the Second World War, there are many living-type satellite cities were built around densely populated metropolis such as Tokyo, Osaka and Nagoya in Japan, for instance, cities Fuchu and Musashino as two satellite cities locate around Tokyo. 3. Public social facilities are simultaneously planning established in satellite cities.

Theoretical researches regarding to satellite cities are quite few. In 60's, some location theorists, Beckman [1958], Dacey [1965, 1966], Parrr [1969] and Beckman & Mcpherson [1970] using different ways to demonstrate that, around a metropolis how much satellite cities should be created. The light they shed on is how to build a macro and efficient city hierarchy system, but the inner city's configuration is ignored.

1-2-2 Subcenters

Subcenters were originally emerged in the middle of 20 century. Although they are also created by city government, they differ from satellite cities in the following aspects: 1. Purpose is different. Unlike satellite cities, subcenters are developed, in most cases, due to increasing citizens' welfare or expanding city size. 2. The size of city where they locate is different. Satellite cities are principally created in metropolis since they play the role of dividing metropolitan function, while subcenters are probably established in any sized city. 3. The sizes of themselves are different. Speaking strictly, the size of satellite city is relative larger than subcenter because the former is planned by central or federal government. 4. The place of creation is different. As shown in Figure 1-1, satellite cities are usually established outside the city, while subcenters are built inside of the city. 5. Independency is different. In satellite cities, residents' activities or both residents and producers' activities does not rely on CBD area, so in this sense, they are independent. While in the case of subcenters, both residential and producers' activities are dependent on CBD, they are non-independent.

Up to now, there are two ways in the study on subcenter, the simplest approach introducing non-central employment into household location models assumes work site location is exogenous and treats space by distance gradients. More complex models permit work site location, wage income, transportation costs and residential location to be jointly determined.

In the first approach, White [1977] described a model, in which two-worker (married) and one-worker households are mixed in the metropolitan area. Employment opportunities exist in both center and suburb. Except married women commute to a predetermined suburb, while others works at CBD. Beside two basic factors: composite goods and housing consumption, leisure time is also introduced in residential utility function. The model is crucially used to examine how the emergency of a new employment center influences urban spatial configuration. In her analysis, it is shown that in housing market equilibrium, two-worker households occupy a ring around suburb, while single-worker households locate in central area and places from outer suburb to the edge of the city. Since urban spatial configuration in her model strongly relies on the value of exogenous variable of wage rate, her analysis shows imperfect. Additionally, systemic comparative static analysis is not performed in her model.

Curran et al. [1982] analyzed this case using polar coordinates. Assume the household has two workers, incurring commuting costs to the city center and to a second work place. The person working in the center commutes $x1$ miles to place of residence, while the second worker works $x3$ miles from the center and commutes $x2$ miles to residential location. Let θ denote the angle between the vectors connecting the center to the residential location and second work place respectively. The distance $x2$ may be expressed in terms of $x1$, $x3$ and θ

using the Law of Cosines, $x2^2 = x1^2 + x3^2 - 2x1x3\cos\theta$. Assume a Cobb-Douglas utility function, $U = H^a X^b L1^c L2^d$, where H stands for land consumption, X composite goods, $L1$ and $L2$, respectively, denote leisure time of first worker and second worker in a household; the two workers ($i = 1,2$) earn wages w_i; incur constant marginal travel costs, t_i; and travel at constant speeds s_i. The bid rent surface may be described in terms of θ:

$$p(x1, x3, \theta) = \beta(Uw_1^c w_2^d)^{-1/a}\left[k(w_1 + w_2) - r_1 x1 - r_2(x1^2 + x3^2 - 2x1x3\cos\theta)^{1/2}\right]^{\alpha/a} \quad (1\text{-}1)$$

where k denotes hours available in a day, $\alpha = a+b+c+d$, $\beta = \left[\frac{a^a b^b c^c d^d}{\alpha^\alpha}\right]^{1/a}$, and $r_i = t_i + \frac{w_i}{s_i}$. If travel costs are greater for the second worker than the first worker, i.e., $r_2 > r_1$, the bid rent curve may be positively sloped along a vector from the center, at a distance inside the suburban employment area. At the points beyond the subcenter, the bid rent gradient will have its usual negative slope. They draw illustrative residential zones for suburban workers but do not derive the shape of those zones; that is, they do not examine the conditions that determine the zones' boundaries. The model also shows the loss in analytic convenience when research space is not defined by one-dimensional gradients.

In Sullivan [1986] aggregative general urban model, all firms' headquarters (office sector) locate at CBD and the sector of manufacturing locates at a given location (SBD), while housing producers are existed in both areas. Within his framework, manufacturing sectors communicate with the CBD by performing face-to-face meeting. Both CBD's and SBD's wage rates are endogenously determined in equilibrium of labor market. The model is crucially used to evaluate governmental zoning policy by assuming that activity of housing producers in SBD is limited by zoning and tax policy. His numerical simulation analysis shows that land use policy decreases total employment in both working areas, production activity shifts from CBD to suburbs, CBD contracts and suburbs expand, and aggregate land rent decreases. For effects on consumers, total wages increases; population density decreases and land rents in unzoned residential areas increase by land use policy.

Sasaki [1990] establishes a similar model, in which both locations of CBD and SBD are also predetermined, and factories in both areas perform production under condition of constant returns to scale and product market is assumed perfect competition. Each firm incurs face-to-face communication with others in both main (CBD) and local (SBD) trading floors. His numerical analysis shows that establishment of a subcenter results in expansion of city size, increases of personal income, firms' profit and residential utility level. When a subcenter is established, the lowering transport cost increases the area for industrial use more than the area for residential use. As the transport cost increases, the number of firms located in CBD increases.

Unlike White [1977], Sullivan [1986] and Sasaki [1990]'s works, Yinger [1992] designs a duo-centric model, in which Anas and Moses [1979]'s transportation system is introduced and the secondary employment center is created at a discrete, landless point. All spoke roads coming out from CBD are assumed as highway. Under the assumption, suburban workers commute along the circular street on which they live to the spoke that goes to the suburb and then commute along that spoke to work site. Complex transportation system and non-general commuting patterns yield four possible urban spatial structures. The urban shape of each urban configuration and the boundary condition of two residential zones for each case are explicitly obtained in his paper. However, Because of complexity of transportation system, in a closed city it is difficult to derive an explicit function for $\bar{u}$, the distance from CBD to the outermost point in the city residential zone. Thus, in order to perform comparative static analysis, his model only sticks to an open city.

In a recent research of Ross & Yinge [1995], along the work performed by Sasaki & Kaiyama [1990], they develop Yinger [1992]'s model by considering an area CBD and SBD. Their model is general in that wage rates are endogenously determined and land input into production is considered. As the same reason as in Yinger [1992], the condition of describing outer urban fringe of SBD area cannot be formulized in a closed city, so the model only takes an open city as its research objective. They attempt to obtain explicit results of comparative static analysis, however, in return for a generality of the model, signs are ambiguous in many cases.

In the second approach, the complexity in introducing decentralized employment opportunities and endogenous work place locations into a general equilibrium model of urban spatial structure arises because bid rent and wage offer gradients are interdependent. Households' bids for land at any given residential location will depend on their work place, wages, and commuting costs; similarly, a household's willingness to accept any given wage at any work site will depend on its residential location, housing prices, and commuting costs.

Solow [1973] first suggested that persons might be employed in a local-goods sector, whose output was produced and sold to households at the location where they reside. Firms have identical production functions defined over capital, land and labor. All firms are initially located at the center but some consider moving out. In his framework, the only gain from such a move is lower land prices in suburbs; while the cost is higher freight transportation charges incurred by suburbanized firms, since all products are assumed to be marketed at the center. He derives bid rent function for firms on the basis of profit maximization over location [see also Moses [1962] and Beckmann [1974]]. In his formulation, time costs are omitted, and hence the wage gradient making households indifferent to work place can be calculated by netting out transport outlays. Other authors have adopted the same approach. In White [1976]'s aggregative model, firms who employ capital, land and labor to perform production make location decision in the trade-off between two coexistent positive and negative efforts: lower wage rates and labor scarcity in suburb. Households who work at the suburb might commute from more central or less central residential locations, and hence the land rent gradient might have local peak at secondary export point. She also examined the effect of establishment of a subcenter on residential welfare and city size by specifying utility function in Cobb-Douglas form. In her analysis results, it is found that subcenter formation increases consumers' welfare and expands city size. However, in her model, the demand for land by firms is not treated explicitly although it is essential for presenting the land use pattern.

Ogawa & Fujita [1980] developed a model of non-monocentric urban land use for relaxing monocentric assumption, in which firms' and residents' locations are not specified a priori. They demonstrate three properties that are necessary in any urban land use equilibrium: 1. Cross commuting of households cannot occur. 2. If commuting takes place, then the equilibrium wage offered in business district must be declining at rate t (commuting cost per unit distance) towards the closest residential district. 3. Business never locates at urban fringe. The equilibrium land use patterns, in their model, are parameterized along t, τ (business firms' transaction cost per unit distance). Their numerical analysis shows that a monocentric pattern dominates if labor commuting costs are small relative to the business firms' transaction costs. Oppositely, when t is relatively large to τ, the city will show a completely mixed land use pattern that firms and residents co-locate any point of the city. The third land use pattern is imperfect mixed one, in which business and residential activities are integrated near the median point 0. Next to this integrated area are pure business districts, followed by pure residential district.

Based on the approach of non-monocentric, Fujita & Ogawa [1982] developed another approach to study multi-centric problem. In their model, the spatial configuration of city is treated as an outcome of interactions between business firms, which perform production under agglomeration economies, and households, which provide labor and follow the employment distribution. Within the framework, boundary conditions of each pattern are derived explicitly and urban land use patterns are also determined in the system of parameters. Besides three land use patterns discussed in Ogawa & Fujita [1980], another two configurations of city are obtained: duo-centric urban configuration and less standard one: tricentric urban configuration. Their work developed a general theory of non-monocentric urban land use, however, the complexity of the emerging possible patterns forbade an analytical solution.

Unlike models cited above, Wieand [1987] designed a duo-centric model in a two-dimensional plane. As same as white [1976]'s, firms use land, capital and labor to produce goods in both CBD and SBD areas, and firms in CBD competes with those in SBD for labor. Since residents commuting to the subcenter by the radial way coming out from the subcenter, the shape of suburb area shows a shape of semicircular. Comparing with White [1976]'s work, he developed the model in that equilibrium of two periods is formulized. In the short run, capital mobility is assumed static and housing supply in each location is constant. With decentralized movement of population, an increase in wage rate forces firms to decentralize for accessing their labor forces so causes a formation of subcenter. In the long run, capital mobility from CBD to SBD is permitted, and housing supply is adjusted to adapt the increasing population in the second residential zone. However, his model cannot conclude a population condition for city workers. Without this condition, his closed model cannot be solved for outer edge of SBD zone.

In White [1988]'s aggregative urban model, she explored the determination of residential and job locations and the pattern of commuting behavior. In her paper, two group residents are classified in terms of skill level they hold. Her analysis shows that workers in any residential ring could potentially be indifferent over commuting journey lengths. That is, extensions of commuting journey caused by changes in workplace location would be compensated by changes in the wage rate, while extensions of the commuting journey lengths caused by changes in residential location would be compensated by lower housing prices.

Some authors suggest that imperfect competitive market should be introduced into model of analyzing urban land use. Fujita and Thisse [1991] analyze a spatial duopoly model with an endogenous residential structure. In their long narrow region, consumers' composite goods are provided by two firms who locate in the region. They jointly carve up the market of composite goods in the area, so their location choice is immediately determined by the length of residential area, and then lead to a variety of urban configurations. Their simulation analysis finds that if residential area is small enough, firms will locate at center together so that urban configuration will be a monocentric city. If market area is relative large, firms' separate location-decision will lead to a duo-centric urban configuration. If residential area is sufficiently large, tow firms' completely separate and regional spatial configuration of two monocentric cities is obtained.

Henderson & Slade [1993] also designed an imperfectly competitive equilibrium to study duo-centric problem. In their paper, city is linear and residents' land consumption is independent of their location so the utility maximization problem is presented as the problem of maximizing residents' composite goods. In their single-stage closed model, two developers who are assumed to be a coalition of residents choose residential locations for maximizing residential utility, while in an open system, developers as profit-maximization agents, determine their business district and labor forces. In the two-stage model, two developers enter the region sequentially, the first entrant who holds first-mover advantage acts strategically to alter the behavior or limit the later entrant. In later model, two cases are analyzed: acquiescing and precluding entry. Analysis for each case is performed numerically. Results shows that in the case of acquiescence, the first mover may use his priority to earn more profits than the second one by building a larger center in or near the central area. While in the case of precluding entry, an enormous development of the first one is required, so as to make the marginal cost of labor sufficiently expensive to an entrant.

1-2-3 Edge cities

Up to now, we have introduced two phenomenon of suburbanization. Recently, a new phenomenon of suburbanization, edge cities are first observed and submitted by Garreau [1991]. In his monograph, five indexes are used to define edge cities (Garreau [1991, P. 6-7]): 1, size of office space, 2, Size of retail space, 3, job opportunities, 4, city function, 5, situation they have ever been thirty years ago. Edge cities are named due to two aspects: 1, they formed at the edge area far from the old metropolitan downtown area and 2, they hold all functions a city ever has.

Formation of edge cities vastly differs from the traditional suburbanization or urban sprawl, the differences are crucially expressed at following aspects. 1. They are planning created by developers or real estate speculators, while traditional subcenters or satellite cities are planned and developed by city or central governments. 2. The purpose is different. In the case of satellite cities or subcenters, they are established to solve or modify social problems occurred in metropolitan areas, such like congestion and population problem. While in the case of edge cities, they are created by developer in order for pursuing maximum profits. For edge cities, developers or speculators make strategic choice of office space capacity, location vis-à-vis the central city, industry /job mix in allocating office space and perhaps population to maximizing their profits. 3. Edge cities' export products are not traditional manufacturing goods. Rather, in edge cities, the offices are the factories of information age. 4. With the emergency of the new employment place, inhabitants might alter their location decision to move outward. A large-scale out-migration might cause a steep decay of core city, finally results in both population and land sizes of edge city exceeding core city, which is unconceivable in the cases of both satellite cities and subcenters.

Henderson and Mitra [1996] pioneered a new approach to study edge cities. In their paper, two models are created: controlled and uncontrolled development. In the model of controlled development, they assume that a big developer who holds a large number of small firms plans the second employment center. In edge city, firms' production activity is influenced by information that is concentrated in core city and decayed with distance. The problem to developer is how to strategically choose edge city location, labor forces and office space capacity vis-a-vis a passive core city to maximize his profit. In equilibrium, three possible urban configurations are obtained, which correspond monocentric, duo-centric and two-monocentric respectively. Through performing numerical simulation, they examined how the office space capacity alters regional spatial structure. The results show that in any region of parameter space, the region configuration tends two separate monocentric cities when city's capacity is in a low level. With port city being strong, edge city will be sucked into the port city business district so as to yield a monocentric configuration. As for the effects of port city's capacity on edge city's capacity, population and developer's profit, it is shown that edge city's capacity does not necessarily decrease with core city's before achieving some threshold, while population in edge city and developer's profit are decreased by increasing port city's capacity. However, the model cannot sufficiently reflect the competition among all user for land because they assume that firms do not demand land and residential land consumption is independent of their locations.

In the recent research of Fujita, Thisse and Zenou [1997], two kinds of behavior of developer are considered. In their first model, developer is assumed as a large firm who holds market power and can anticipate the impacts of his location decision on labor market. While in their later model, developer is treated as an agent who engages in both activities of production and real estate. According to developer's location choice, three land use patterns are obtained: monocentric, duo-centric and two separate monocentric. Their model is more general than Henderson et al work in that wage rate is endogenously determined, although labor demand function is specified.

Up to now, three phenomenon of suburbanization have been discussed in the chapter, in order to distinguish the three concepts, we summarize the differences among them in Table 1-1.

1-3 Organization of the paper

With the boom of the process of suburbanization, some questions are submitted by the new phenomenon to urban economists, for instance:

1. How does the process influence the behavior of each agency in the city?
2. Does suburbanization bring us good or bad? Or on the other word, who is largest beneficial from the process of suburbanization?
3. With the development of suburb, will central area demolish or disappear?
4. How does the suburbanization affect urban spatial structure?

Based on traditional monocentric model, this paper establishes two theoretical duo-centric models to answer the above questions. The configuration of the paper is organized as following:

Chapter two describes a closed duo-centric urban model, in which Anas and Moses [1979]' transportation system is introduced. Under such transportation system of including both an infinite radial lines and a dense circular roads, what urban shape and spatial configuration will be yielded. Furthermore, we perform comparative static analysis on it and compare the results obtained with those in Wheaton [1974] (see also Pines and Weiss [1976], Sasaki [1987a] and Brueckner [1987])'s work to examine that whether an establishment of a subcenter alter properties obtained in a monocentric city. Meanwhile, using analytic results, we will reply the questions submitted above.

	Satellite cities	Subcenters	Edge cities
Purpose of creation	Burden metro function or solve social problem occurred in metropolis	Enhance social welfare or expanding urban scale	Pursuit maximum profit
Agent of creation	Central or federal Government	City Government	Developers or Speculators
Independency	Independent	Non-independent	Independent
Size	Largest	Smallest	Between satellite city and subcenter
Size of city where it is located	Metropolis	Any sized city	Any sized city
Location of creation	Somewhere around edge of city	Inside of city	Outside of city

Table 1-1

Chapter three presents an open duo-centric model, in which the same transportation system is introduced. In the section, subcenter location, as an important index, will be evaluated, i.e., in what scope subcenter location alters, the researching region shows duo-centric or two monocentric spatial structure. Based on analyses in Zhang & Sasaki [1997], we also carry out comparative static analysis on this model and take comparison with the results obtained in analyzing monocentric city to examine that whether a establishment of subcenter change effects of parameters or not.

Chapter four creates a model to study edge cities, in which we consider two equilibrium of both land and labor markets to examine that how a creation of an edge city affects regional spatial configuration. In the section, we also perform a comparison with Henderson & Mitra [1996]'s and Fujita, Thisse & Zenou [1997]'s work.

In chapter five, we carry out a retrospect on the works in the paper and in the last section of the chapter, we discuss some approaches for improving our models.

Chapter 2

Model of a closed city with a subcenter

2-1 Introduction

Traditional monocentric model is usually classified as closed city and open city models. In the closed-city model, the population of the city is given exogenous, while utility level is endogenously determined in the system. With the boom of researching suburbanization, these two kinds of models are cited. So far, most of literature (see White [1976, 1978, 1988], Sullivan [1986]) about non-central employment follows the approach of assuming non-central worksite location being established on a circular highway running around the CBD and treats space by distance gradients. By this assumption, the shape of city keeps previous circular character, so all approaches in Alonso-type [1964] monocentric city are still suitable in analyzing duo-centric urban configuration. For instance, White [1976] first examined the effect of establishing a subcenter on utility level and city size. By assuming that subcenter is created in a certain ring far from CBD, She found that both consumer welfare and city size increase where a subcenter is established in a closed city, although the utility function was specified in Cobb-Douglas form. The effect on total land value is ambiguous and comparative static analysis was not developed in her paper.

In Sullivan's model [1986], the office sector and manufacturing sector respectively locate in CBD and suburbs, and the building-height in CBD is restricted by land-use policy characterized by the capital-land ratio. His simulation result shows that land use policy decreases total employment in both working areas, production activity shifts from CBD to suburbs, CBD contracts and suburbs expands, and aggregate land rent decreases. For effects on consumers, total wages increase, population density decreases and land rents in unzoned residential areas increase due to land use policy.

Sasaki [1990] designs two trading floors in terms of the quality of information and the function of a firm, in which the local trading floor was defined as a subcenter. His numerical analysis shows that personal income (endogenously determined), profit of a firm, utility of a resident, and the share of the rental redistribution in personal income are increased by establishing a subcenter. In the examination of the effect of transportation cost on city size, he found that the city is expanded by decreasing commuting cost in both monocentric city and duo-centric city cases.

Unlike the models cited above, some literature suggests that it is more general or realistic to assume the subcenter location as a discrete point. Yinger [1992] designs a transportation network (originally submitted by Anas and Moses [1979]) including an infinite number of spoke-roads and dense circle streets in a city and subcenter is created on only a point far from the CBD. However, complexity of transportation system and more possible land use patterns make the model be intractable.

In the recent work of Ross and Yinger(1995), they perform a challenge on this complex urban system. Their model is general in that wage rates are endogenously determined and land input into production is considered. They attempt to obtain explicit results of comparative static analysis, however, in return for the 'generality' of a model, the sign is ambiguous in many cases: only the effect of a change in agriculture land rent is clearly determined. The effect of transportation cost change is a great concern to urban-policy makers, but its effect on population size, for instance, is ambiguous. Such 'indeterminacy' in a two-center city is predictable. This is because even in a monocentric-model, whether a closed or an open city model, the comparative static results are ambiguous, if endogenous wage rates and land consumption of industry are incorporated. (See Sasaki and Kaiyama [1990]).

While Ross and Yinger [1995] have focused on an open city, this paper intends to perform comparative static analysis in the setting of a closed, two-center city so as to evaluate the effects of establishing a subcenter. Based on a lesson from the works so far, we assume the exogenous wage rates and no land consumption of industry in order to obtain unambiguous results. Needless to say, it is restrictive to assume that firms do not demand land as an input into production, and the labor income is independent of the spatial distribution of employment. Recognizing this limitation, the present paper intends to analyze, as a first approximation, the effects of subcenter formation in a city. From the results of comparative static analysis, it is found that the establishment of a subcenter keeps the most important properties obtained in a monocentric city unchanged.

The model described in the second section is based on the earlier urban general equilibrium model, which employs Anas and Moses [1979]'s transportation system. We focus on a closed city model. Comparative static analysis is developed in section 3 by employing Sasaki's [1987] approach, and some main conclusions are summarized in the last section.

2-2 Model

2-2-1 Model of monocentric closed city

Assume that there is an Alonso-type city, a plain including an export node in the center that is called a CBD. We assume that the CBD as an employment center does not occupy any land. Consumers with the same tastes reside around the CBD and earn the same wage income by working there. The utility function is assumed to be strictly quasiconcave, where the utility level depends only on two normal goods: composite good x and residential land size q, namely

$$u = u(x, q). \tag{2-1}$$

We assume that the urban transportation system consists of not only an infinite number of spoke-roads coming out from the CBD, but an infinite number of circle streets running around the CBD. A household will maximize its utility level (1), under its budget constraint

$$y = x + rq + kt. \tag{2-2}$$

where y is income level, r market land rent, k commuting cost of round-trip per mile, t the location of a household, and the price of x is defined numeraire. The utility level obtained in equilibrium should be the same among consumers regardless of their location. Absentee landlords, as land owners, will rent their land to the highest bidder, and therefore the market land rent must coincide with the bid rent of a consumer associated with the utility level attained in equilibrium, that is

$$r(t) = \max_{x,q} \frac{y - x - kt}{q}$$

$$s.t. \quad u = u(x, q). \tag{2-3}$$

Solving this problem, we obtain the optimal value of x, q and r.

$$x = x(u,t,k,y)$$

$$q = q(u,t,k,y) \qquad (2\text{-}4)$$

$$r = r(u,t,k,y).$$

By using the envelope theorem, the results of comparative statics will be

$$\frac{\partial r}{\partial u} = -\frac{1}{u_x q} < 0, \quad \frac{\partial r}{\partial y} = \frac{1}{q} > 0, \quad \frac{\partial r}{\partial t} = -\frac{k}{q} < 0, \quad \frac{\partial r}{\partial k} = -\frac{t}{q} < 0. \qquad (2\text{-}5)$$

The equilibrium conditions of a closed city are shown in the following equations

$$r(u,b,k,y) = S. \qquad (2\text{-}6)$$

$$2\pi\int_0^b \frac{t}{q(u,t,k,y)}dt = N. \qquad (2\text{-}7)$$

or

$$Sb - \int_0^b r(u,t,k,y)dt = -\frac{Nk}{2\pi}. \qquad (2\text{-}7')$$

where S is rural land rent, b the urban fringe distance and N expresses the population in the city. In the case of a closed city, utility level and urban fringe are determined endogenously in the system of (2-6) and (2-7).

2-2-2 Model of a closed city with a subcenter

Now we assume that a city government establishes another export node at a certain point (point A in Figure 2-1) in the existing monocentric city, which is m kilometers from the CBD. Location behavior of firms is not considered in this model, but we just assume that location A becomes a new employment center (subcenter) and a sufficient number of firms locate there so as to employ all people who want to work there. Moreover, it is assumed for simplifying the analysis that firms in the subcenter offer the same wage level as those in the CBD and the transport cost for commuting unit distance is the same at k for both workplaces. Since a subcenter (SBD) is established at a certain point, the new urban shape will not be circular any more, and the bid rent curve radiated from the CBD will not be the same in all directions. The existence of a new employment center causes bid rents around the SBD to rise due to saved commuting costs. Consequently, the equilibrium of the old urban spatial structure will be distorted, and new equilibrium will not be attained until the utility level becomes the same regardless of residential locations. We separate the city into two parts in accordance with two worksites. As shown in Figure 2-1, we define the area occupied by residents who commute to the CBD and the SBD, respectively, as the CBD area and the SBD area[1]. Thus, the key feature of the new urban structure will be urban shape and the boundary between the two groups of workers. It is found that workers who reside in the CBD area still commute to the CBD by spoke-road, while those living in the SBD area (except for those living in the spoke or circle street that intersect the subcenter) commute to the SBD by two methods according to location: if the resident's distance from the CBD is less than the subcenter's (if he is closer), he reaches the spoke through the subcenter by the circle street via his location first, then goes to the SBD by that spoke; if the resident's distance from the CBD is greater than the subcenter's (if he is further away), he will get to the circle street through the subcenter by the spoke through his location first, and then reach the SBD by that circle street. Between two residential zones, there must be a demarcation line on which consumers incur the same commuting cost regardless of their worksite. We divide this demarcation line into two sections: the section closer than the subcenter is defined as the inner-demarcation, and the one further than the subcenter is defined as the outer-demarcation. Now let us determine this demarcation line. In Figure 2-1, on each spoke road between the vertical spoke and the spoke which forms 1 radian with the vertical spoke, there exists one and only one point on which consumers incur the same transportation distance regardless of their worksite. If we link these points from corresponding sectors, it will form an inner-demarcation line, i.e., EF or EF' curve. We denote the inner-demarcation line by $v(\theta)$, and its mathematical expression will be

[1] The Shapes of equilibrium urban configuration in the setting of two centers have already been drawn in Yinger[1992]. Figures in the present paper are shown just for convenience of explanation.

$$v(\theta) = \frac{m}{2-\theta} \qquad \theta \in [0,1]. \tag{2-8}$$

where θ is the radian formed by the vertical spoke and any other spoke. $\theta = 1$ means that arc AF=OF, which is obtained by $\theta m = m$. A consumer living in the area where θ is greater than 1 will not commute to the SBD, the distance of commuting there being greater than commuting to the CBD. Thus the outer-demarcation line will not vary with θ, but will coincide with the extension line of the spoke through $v(1)$, as line FG or F'G' in Figure 2-1.

The bid rent of a consumer who lives in the CBD area can be obtained from (2-3), and the bid rent of a consumer living in the SBD area is defined as

$$R(t,\theta) = \max_{X,Q} \frac{y - X - k\left[|t-m| + \theta \min(t,m)\right]}{Q} \qquad \theta \in [0,1]$$

$$s.t. \quad u = u(X,Q). \tag{2-9}$$

where X and Q respectively denote the consumption of composite good and land consumption of a resident in SBD area.

Solving this problem, the optimal values of X, Q and R will be expressed as

$$\begin{aligned} X &= X(u,t,k,y,m,\theta) \\ Q &= Q(u,t,k,y,m,\theta) \\ R &= R(u,t,k,y,m,\theta). \end{aligned} \tag{2-10}$$

The comparative static results concerning (2-10) are obtained by applying the envelope theorem:

$$\frac{\partial R}{\partial u} = -\frac{1}{u_X Q} < 0, \qquad \frac{\partial R}{\partial y} = \frac{1}{Q} > 0, \qquad \frac{\partial R}{\partial k} = -\frac{|t-m| + \theta\, min(t,m)}{Q} < 0, \tag{2-11}$$

$$\frac{\partial R}{\partial t}=\begin{cases}-\dfrac{k}{Q}<0, & t>m\\ \dfrac{k(1-\theta)}{Q}>0, & t\le m\end{cases},\quad \frac{\partial R}{\partial m}=\begin{cases}-\dfrac{k}{Q}<0, & m>t\\ \dfrac{k(1-\theta)}{Q}>0, & m\le t\end{cases},\quad \frac{\partial R}{\partial \theta}=-\frac{k\,min(t,m)}{Q}<0.$$

We let L denote the CBD-urban fringe that corresponds to the CBD area, i.e., arc GG' (solid line) and let H denote the SBD-urban fringe that corresponds to the SBD area, i.e., line WG or WG'. Once L is known, H will be determined by the following formula:

$$H(\theta)=L+(1-\theta)m \qquad \theta\in[0,1]. \tag{2-12}$$

However, the new urban shape will be different according to the new worksite's position. In terms of distance to CBD, CBD-urban fringe can be closer or farther than subcenter location, so we have to discuss both situations separately.

Case A

In case A, $L_A>m$. A typical urban shape is shown in Figure 2-2. In this new equilibrium, we have three variables: u, L_A and H_A, which will be simultaneously determined by the following three equations:

$$r(u,L_A,k,y)=S. \tag{2-13}$$

$$H_A(\theta)=L_A+(1-\theta)m \qquad \theta\in[0,1]. \tag{2-14}$$

$$(2\pi-2)\int_0^{L_A}\frac{t}{q}dt+2\int_0^{\frac{m}{2}}\int_0^1\frac{t}{q}d\theta dt+2\int_{\frac{m}{2}}^{m}\int_0^{1-\left(2-\frac{m}{t}\right)}\frac{t}{q}d\theta dt+2\int_{\frac{m}{2}}^{m}\int_0^{2-\frac{m}{t}}\frac{t}{Q}d\theta dt$$
$$+2\int_m^{L_A}\int_0^1\frac{t}{Q}d\theta dt+2\int_{L_A}^{m+L_A}\int_0^{1-\frac{t-L_A}{m}}\frac{t}{Q}d\theta dt=N \tag{2-15}$$

or

$$2\pi\left(SL_A-\int_0^{L_A}rdt\right)-4\int_0^{\frac{m}{2}}rdt+4\int_{\frac{m}{2}}^{m}rdt+Sm-2\int_0^m\frac{t}{m}rdt=-kN. \tag{2-15'}$$

Relation (2-13) means that the bid rent at the CBD-urban fringe should be equal to rural land rent.

Equation (2-14) expresses the relation between two urban fringes. Relation (2-15) or its alternative form (2-15') represents the population condition of new urban structure. In (2-15), the first three terms on the left side denotes the population in the CBD area, the fourth part and the sum of fifth and sixth parts respectively express the number of people whose location is less and greater than subcenter location in the SBD area. Derivation of (2-15') is depicted in the mathematics appendix.

Case B

In case B, $L_B < m$. The urban shape of this case is depicted in Figure 2-3. The urban shape in this case is somewhat complicated. We determine first the urban shape and the boundary between residential areas. In Figure 2-3, since L_B is less than m, the arc of the CBD-urban fringe must intersect with the inner-demarcation line at some point (e.g., at point D or D'), we can draw an Iso-bid rent curve with θ until it intersects with the circle street via the SBD at point G or G', on which the bid rent is equal to S. Point W is easily determined by adding L_B from point A along the vertical bar, and we connect W and G or G' by an Iso-bid rent curve on which rent is equal to agricultural land rent. The urban shape will be depicted as the thickest outline in Figure 2-3. In this case, there is no outer-demarcation line and the SBD-urban fringe includes two parts: we define, respectively, the part less than subcenter location as inner-SBD-urban fringe E and that greater than subcenter location as outer-SBD-urban fringe H_B. Defining $\angle AOD$ in Figure 2-3 as θ_1 and $\angle AOG$ as θ_2, their mathematical expressions are:

$$\theta_1 = 2 - \frac{m}{L_B}. \tag{2-16}$$

$$\theta_2 = \frac{L_B}{m}. \tag{2-17}$$

In this system, we have four determinants u, L_B, E and H_B, which are jointly determined by the equilibrium conditions in (2-18) through (2-21) (or (2-21')).

$$r(u, L_B, k, y) = S. \tag{2-18}$$

$$E(\theta) = \frac{m - L_B}{1 - \theta} \qquad \theta \in [\theta_1, \theta_2]. \tag{2-19}$$

$$H_B(\theta) = L_B + (1 - \theta)m \qquad \theta \in [0, \theta_2]. \tag{2-20}$$

$$2(\pi - \theta_1)\int_0^{L_B} \frac{t}{q} dt + 2\int_0^{\frac{m}{2}} \int_0^{\theta_1} \frac{t}{q} d\theta dt + 2\int_{\frac{m}{2}}^{L_B} \int_0^{\theta_1 - \left(2 - \frac{m}{t}\right)} \frac{t}{q} d\theta dt + 2\int_{\frac{m}{2}}^{L_B} \int_0^{2 - \frac{m}{t}} \frac{t}{Q} d\theta dt$$

$$+2\int_{L_B}^{m} \int_0^{1 - \frac{m - L_B}{t}} \frac{t}{Q} d\theta dt + 2\int_m^{m + L_B} \int_0^{1 - \frac{t - L_B}{m}} \frac{t}{Q} d\theta dt = N. \tag{2-21}$$

or

$$(2\pi - 4)\left(SL_B - \int_0^{L_B} r dt\right) - 8\int_0^{\frac{m}{2}} r dt - 2\int_0^{L_B} \frac{t}{m} r dt + 4mS + \frac{S(L_B)^2}{m} = -kN. \tag{2-21'}$$

Relations in (2-18), (2-19), and (2-20), respectively denote three urban fringe conditions in equilibrium, while equation (2-21) (or (2-21')) denotes the population condition in the new urban structure. In (2-21), the first three parts on the left side express: the CBD area population; the sum of fourth, fifth and sixth parts denotes the number of people with location closer than subcenter location in the SBD area; and the last term shows the population with residential location further than subcenter location in the SBD area.

Between case A and B, there exists a boundary case in which the CBD-urban fringe is equal to subcenter location. The urban shape for this case is described in Figure 2-4. We define this boundary location as m'. The utility level u, the CBD-urban fringe L', and the SBD-urban fringe H' under this special case will be obtained by:

$$r(u, L', k, y) = S. \tag{2-22}$$

$$H'(\theta) = 2L' - \theta m' \qquad \theta \in [0, 1]. \tag{2-23}$$

$$(2\pi - 2)\int_0^{L'} \frac{t}{q} dt + 2\int_0^{\frac{L'}{2}} \int_0^{1} \frac{t}{q} d\theta dt + 2\int_{\frac{L'}{2}}^{L'} \int_0^{\frac{L'}{t} - 1} \frac{t}{q} d\theta dt$$

$$+2\int_{\frac{L'}{2}}^{L'}\int_0^{2-\frac{L'}{t}}\frac{t}{Q}d\theta dt+2\int_{L'}^{2L'}\int_0^{2-\frac{L'}{t}}\frac{t}{Q}d\theta dt=N\,. \tag{2-24}$$

or

$$2\pi\left(SL'-\int_0^{L'}rdt\right)-4\int_0^{\frac{L'}{2}}rdt+4\int_{\frac{L'}{2}}^{L'}rdt+SL'-2\int_0^{L'}\frac{t}{L'}rdt=-kN\,. \tag{2-24'}$$

It is easily predicted that if a subcenter is established at a location very far from the CBD, the initial monocentric city will be divided into two cities, and the subcenter will become the CBD of a new city. Urban shape of this case is figured in Figure 2-5. Therefore, there exists a maximum of m in case B, beyond which there will exist rural land between the CBD area and the SBD area. Denoting the maximum location, the CBD-urban fringe and two SBD-urban fringes, respectively, by m^*, L^*, E^*, and H^* in this case, it is seen that $m^*=2L^*$, $\theta_1=0$ and $\theta_2=\frac{1}{2}$. Four variables are determined by

$$r(u,L^*,k,y)=S\,. \tag{2-25}$$

$$E^*(\theta)=\frac{L^*}{1-\theta} \qquad \theta\in\left[0,\frac{1}{2}\right]. \tag{2-26}$$

$$H^*(\theta)=3L^*-2\theta L^* \qquad \theta\in\left[0,\frac{1}{2}\right]. \tag{2-27}$$

$$2\pi\int_0^{L^*}\frac{t}{q}dt+2\int_{L^*}^{m^*}\int_0^{1-\frac{L^*}{t}}\frac{t}{Q}d\theta dt+2\int_{m^*}^{3L^*}\int_0^{\frac{3L^*-t}{2L^*}}\frac{t}{Q}d\theta dt=N\,. \tag{2-28}$$

or

$$2\pi\left(SL^*-\int_0^{L^*}rdt\right)-4\int_0^{L^*}rdt-\int_0^{L^*}\frac{t}{L^*}rdt+\frac{9}{2}SL^*=-kN\,. \tag{2-28'}$$

2-3 Comparative Static Analysis

Before starting the analysis, we prepare some useful mathematical relations below. Under the assumptions of a utility function and a household behavior described above, we see that:

$$\frac{\partial x}{\partial t} < 0, \qquad \frac{\partial q}{\partial t} > 0. \tag{2-29}$$

Furthermore, it holds that

$$\frac{\partial u_x}{\partial t} > 0$$

$$\frac{\partial^2 r}{\partial u \partial t} = \frac{\frac{\partial u_x}{\partial t} q + \frac{\partial q}{\partial t} u_x}{(u_x q)^2} > 0. \tag{2-30}$$

If $\psi(t)$ is a decreasing continuous function of t and $t \in [0, \beta]$, then the following inequality holds:

$$\int_0^{\frac{\beta}{2}} \frac{\psi}{\beta - t} dt > \int_{\frac{\beta}{2}}^{\beta} \frac{\psi}{t} dt > \int_{\frac{\beta}{2}}^{\varepsilon} \frac{\psi}{t} dt \tag{2-31}$$

where $\varepsilon \in \left[\frac{\beta}{2}, \beta\right]$.

2-3-1 Analysis for case A

For the effect of changes in k, the procedure of the comparative static analysis is as follows: Equation (2-13) is differentiated with respect to k and by using (2-5) we obtain:

$$-\frac{1}{u_x(L_A)} \frac{du}{dk} - k \frac{dL_A}{dk} = L_A \ . \tag{2-32}$$

Equation (2-14) is differentiated with respect to k to give:

$$\frac{dH_A}{dk} = \frac{dL_A}{dk} . \tag{2-33}$$

Equation (15') is also differentiated with respect to k to give:

$$\left[2\pi\int_0^{L_A}\frac{1}{u_x q}dt+4\int_0^{\frac{m}{2}}\frac{1}{u_x q}dt-4\int_{\frac{m}{2}}^{m}\frac{1}{u_x q}dt+2\int_0^{m}\frac{t}{m}\frac{1}{u_x q}dt\right]\frac{du}{dk}$$

$$=-N-2\pi\int_0^{L_A}\frac{t}{q}dt-4\int_0^{\frac{m}{2}}\frac{t}{q}dt+4\int_{\frac{m}{2}}^{m}\frac{t}{q}dt-2\int_0^{m}\frac{t}{m}\frac{t}{q}dt. \tag{2-34}$$

Solving (2-34) for $\dfrac{du}{dk}$ and employing (2-29)-(2-31), we have:

$$\frac{du}{dk}=-\frac{N+2\pi\int_0^{L_A}\frac{t}{q}dt+4\int_0^{\frac{m}{2}}\frac{t}{q}dt-4\int_{\frac{m}{2}}^{m}\frac{t}{q}dt+2\int_0^{m}\frac{t}{m}\frac{t}{q}dt}{2\pi\int_0^{L_A}\frac{t}{u_x q}dt+4\int_0^{\frac{m}{2}}\frac{t}{u_x q}dt-4\int_{\frac{m}{2}}^{m}\frac{t}{u_x q}dt+2\int_0^{m}\frac{t}{m}\frac{t}{u_x q}dt}<0. \tag{2-35}$$

Substituting (2-35) into (2-31) and (2-32), we have:

$$\frac{dH_A}{dk}=\frac{dL_A}{dk}\gtrless 0. \tag{2-36}$$

This ambiguous effect will be cleared in the later by using different way.

To find the effects of changes in y, the same procedure was applied with relations (2-29) and (2-30):

$$\frac{du}{dy}=\frac{2\pi\int_0^{L_A}\frac{1}{q}dt+4\int_0^{\frac{m}{2}}\frac{1}{q}dt-4\int_{\frac{m}{2}}^{m}\frac{1}{q}dt+2\int_0^{m}\frac{1}{q}dt}{2\pi\int_0^{L_A}\frac{1}{u_x q}dt+4\int_0^{\frac{m}{2}}\frac{1}{u_x q}dt-4\int_{\frac{m}{2}}^{m}\frac{1}{u_x q}dt+2\int_0^{m}\frac{1}{u_x q}dt}>0. \tag{2-37}$$

$$\frac{dH_A}{dy}=\frac{dL_A}{dy}=\frac{\left[\begin{array}{l}2\pi\int_0^{L_A}\frac{u_x(L_A)-u_X}{u_xq}dt+4\int_0^{\frac{m}{2}}\frac{u_x(L_A)-u_X}{u_xq}dt\\ -4\int_{\frac{m}{2}}^{m}\frac{u_x(L_A)-u_X}{u_xq}dt+2\int_0^{m}\frac{t}{m}\frac{u_x(L_A)-u_X}{u_xq}dt\end{array}\right]}{\left[\begin{array}{l}2\pi\int_0^{L_A}\frac{u_x(L_A)k}{u_xq}dt+4\int_0^{\frac{m}{2}}\frac{u_x(L_A)k}{u_xq}dt\\ -4\int_{\frac{m}{2}}^{m}\frac{u_x(L_A)k}{u_xq}dt+2\int_0^{m}\frac{t}{m}\frac{u_x(L_A)k}{u_xq}dt\end{array}\right]}>0. \tag{2-38}$$

For the effect of changes in N: Differentiating (2-13), (2-14) and (2-15') with respect to N, and slightly arranging them, we obtain:

$$\frac{du}{dN}=-\frac{k}{2\pi\int_0^{L_A}\frac{1}{u_xq}dt+4\int_0^{\frac{m}{2}}\frac{1}{u_xq}dt-4\int_{\frac{m}{2}}^{m}\frac{1}{u_xq}dt+2\int_0^{m}\frac{t}{m}\frac{1}{u_xq}dt}<0. \tag{2-39}$$

$$\frac{dH_A}{dN}=\frac{dL_A}{dN}=\frac{1}{2\pi\int_0^{L_A}\frac{1}{u_xq}dt+4\int_0^{\frac{m}{2}}\frac{1}{u_xq}dt-4\int_{\frac{m}{2}}^{m}\frac{1}{u_xq}dt+2\int_0^{m}\frac{t}{m}\frac{1}{u_xq}dt}>0. \tag{2-40}$$

As for the effect of S, from the three basic equations we obtain:

$$\frac{du}{dS}=-\frac{2\pi L_A+m}{2\pi\int_0^{L_A}\frac{1}{u_xq}dt+4\int_0^{\frac{m}{2}}\frac{1}{u_xq}dt-4\int_{\frac{m}{2}}^{m}\frac{1}{u_xq}dt+2\int_0^{m}\frac{t}{m}\frac{1}{u_xq}dt}<0. \tag{2-41}$$

$$\frac{dH_A}{dS}=\frac{dL_A}{dS}=-\frac{\left[\begin{array}{l}2\pi\int_0^{L_A}\left(\frac{u_x(L_A)q(L_A)}{u_xq}-1\right)dt+2\int_0^{m}\frac{t}{m}\left(\frac{u_x(L_A)q(L_A)}{u_xq}-1\right)dt\\ +4u_x(L_A)q(L_A)\left(\int_0^{\frac{m}{2}}\frac{1}{u_xq}dt-\int_{\frac{m}{2}}^{m}\frac{1}{u_xq}dt\right)\end{array}\right]}{2\pi\int_0^{L_A}\frac{1}{u_xq}dt+4\int_0^{\frac{m}{2}}\frac{1}{u_xq}dt-4\int_{\frac{m}{2}}^{m}\frac{1}{u_xq}dt+2\int_0^{m}\frac{t}{m}\frac{1}{u_xq}dt}<0. \tag{2-42}$$

In a similar way, we can analyze the effect of subcenter location. We have:

$$\frac{du}{dm}=\frac{3\int_{\frac{m}{2}}^{m}\frac{k}{q}dt+\int_{\frac{m}{2}}^{L_A}\frac{k}{q}dt-\int_{0}^{m}\frac{t^2}{m^2}\frac{k}{q}dt}{2\pi\int_{0}^{L_A}\frac{1}{u_x q}dt+4\int_{0}^{\frac{m}{2}}\frac{1}{u_x q}dt-4\int_{\frac{m}{2}}^{m}\frac{1}{u_x q}dt+2\int_{0}^{m}\frac{t}{m}\frac{1}{u_x q}dt}\gtreqless 0. \tag{2-43}$$

$$\frac{dL_A}{dm}=-\frac{3\int_{\frac{m}{2}}^{m}\frac{k}{q}dt+\int_{\frac{m}{2}}^{L_A}\frac{k}{q}dt-\int_{0}^{m}\frac{t^2}{m^2}\frac{k}{q}dt}{2\pi\int_{0}^{L_A}\frac{u_x(L_A)k}{u_x q}dt+4\int_{0}^{\frac{m}{2}}\frac{u_x(L_A)k}{u_x q}dt-4\int_{\frac{m}{2}}^{m}\frac{u_x(L_A)k}{u_x q}dt+2\int_{0}^{m}\frac{t}{m}\frac{u_x(L_A)k}{u_x q}dt}$$

$$=-\frac{1}{u_x(L_A)q}\frac{du}{dm}\gtreqless 0 \tag{2-44}$$

$$\frac{dH_A}{dm}=\frac{dL_A}{dm}+(1-\theta)\gtreqless 0. \tag{2-45}$$

Taking advantage of the results so far, it is possible to derive the effects on the market land rents of both residential areas by using (2-5) and (2-11), i.e.

$$\frac{\partial r(0)}{\partial k}>0,\quad \frac{\partial r(0)}{\partial y}>0,\quad \frac{\partial r}{\partial N}>0,\ \frac{\partial r}{\partial S}>0 \text{ and } \frac{\partial r}{\partial m}=-\frac{1}{u_x q}\frac{du}{dm}\gtreqless 0$$

$$\frac{\partial R_{(t=m,\theta=0)}}{\partial k}>0\quad \frac{\partial R_{(t=m,\theta=0)}}{\partial y}>0\quad \frac{\partial R}{\partial N}>0\quad \frac{\partial R}{\partial S}>0 \text{ and}$$

$$\frac{\partial R}{\partial m}=\begin{cases}-\dfrac{1}{u_X Q}\dfrac{du}{dm}-\dfrac{k}{Q}\gtreqless 0, & t<m\\ -\dfrac{1}{u_X Q}\dfrac{du}{dm}+\dfrac{k(1-\theta)}{Q}\gtreqless 0, & t>m\end{cases}. \tag{2-46}$$

Using the results thus far, we can determine the sign of ambiguous effect $\frac{dL_A}{dk}$ in (2-36).

Assume transportation cost is increased from k_0 to k_1. Suppose that both CBD and SBD urban fringes expand as a result of increased transport cost, i.e., $\frac{dH_A}{dk} = \frac{dL_A}{dk} > 0$. In this circumstance, one of the following two situations occurs:

(I) the new rent is higher than the old one at every location.

(ii) the new rent curve intersects with the old one *from below*.

However, both of them are impossible. Situation (i) can not occur since the demand for land would be decreased everywhere while the city is expanded. Situation (ii) is impossible since at the intersection of two curves, the new curve must be steeper (that is, the new curve must intersect the old one *from above*) because less land be demanded and k is larger ($\left|\frac{k}{q}\right|$ gets larger). Thus, it is concluded that $\frac{dL_A}{dk} = \frac{dH_A}{dk} < 0$, namely, a city shrinks when k is increased.

The effect of subcenter location seems ambiguous as far as the equation (2-43) through (2-46) are concerned. However, it is intuitively obvious that the utility level increases with increasing subcenter's location distance from the CBD. This is because the area between CBD and a subcenter is expanded, and some people in the fringe districts of both CBD and SBD areas can save their commuting costs by moving into this expanded central area closer to workplaces, and as a result, can increase their utility level. Actually, it is possible to prove this rigorously in the following way:

From (2-44), we have:

$$\frac{dL_A}{dm} = -\frac{1}{u_x(L_A)q}\frac{du}{dm}. \tag{2-47}$$

First suppose that $\frac{dL_A}{dm} > 0$, then $\frac{du}{dm} < 0$. In this situation, from (2-46), it holds that:

$$\frac{dr}{dm} = -\frac{1}{u_x q}\frac{du}{dm} > 0, \text{ and } \frac{dq}{dm} = \frac{dq}{du}\frac{du}{dm} < 0. \tag{2-48}$$

$$\frac{dN_{CBD}}{dm} = -\frac{1}{k}\left[(2\pi-2)\int_0^{L_A}\frac{1}{u_x q}dt - 2\int_0^{\frac{m}{2}}\frac{1}{u_x q}dt + 2\int_{\frac{m}{2}}^{m}\frac{1}{u_x q}dt\right]\frac{du}{dm} + 2\int_{\frac{m}{2}}^{m}\frac{1}{q}dt > 0. \qquad (2\text{-}49)$$

where (2-48) implies that residential lot size everywhere in the CBD area gets smaller due to a further subcenter location, and (2-49) implies that population in the CBD area is increased (and thus population in the SBD area is decreased) by further subcenter location.

Differentiating (2-8) with respect to m, we obtain:

$$\frac{dv}{dm} = \frac{1}{2-\theta} > 0 \qquad \theta \in [0,1]. \qquad (2\text{-}50)$$

From (2-45), we have:

$$\frac{dH_A}{dm} = \frac{dL_A}{dm} + (1-\theta) > 0 \qquad \theta \in [0,1]. \qquad (2\text{-}51)$$

From (2-50) and (2-51), it is concluded that when a subcenter is established at a more distant place, the SBD area expands in every direction. This implies that any location in the previous SBD area with any distance to the previous subcenter can be found in the new SBD area. In this circumstance, everyone in the present SBD area can always find his (or her) location that has the same commuting distance as before. Since income level, commuting distance and commuting cost of a residence are unchanged and utility level is decreased, residential land rent must be increased everywhere and therefore residential lot size everywhere in the new SBD area must be decreased. Thus population in the SBD area must increase to fully use the expanded SBD area. But this contradicts the fact that SBD population decreases because of (2-49). We thus conclude that $\frac{dL_A}{dm} > 0$ can not hold, and it must be that $\frac{dL_A}{dm} < 0$, and therefore $\frac{du}{dm} > 0$. To summarize, the following are concluded in case A.

$$\frac{dL_A}{dm} < 0 \text{ and } \frac{dL_A}{dm} > -1. \qquad (2\text{-}52)$$

$$\frac{du}{dm} > 0. \tag{2-53}$$

$$\frac{dr}{dm} < 0. \tag{2-54}$$

$$\frac{dH_A}{dm} = \begin{Bmatrix} 1 + \dfrac{dL_A}{dm} > 0, & \theta = 0. \\ \dfrac{dL_A}{dm} < 0, & \theta = 1. \end{Bmatrix}. \tag{2-55}$$

$$\frac{dR}{dm} = \begin{Bmatrix} -\dfrac{1}{u_X Q}\dfrac{du}{dm} - \dfrac{k}{Q} < 0, & t < m. \\ -\dfrac{1}{u_X Q}\dfrac{du}{dm} - \dfrac{k(1-\theta)}{Q} \gtrless 0, & t > m. \end{Bmatrix}. \tag{2-56}$$

That is, in case A, the utility of city residents is necessarily increased and the CBD area necessarily gets smaller as the location of a subcenter becomes farther from CBD. The results for case A are summarized in Table 2-1.

2-3-2 Analysis for case B

The procedure for the analysis for case B is the same as the above. Differentiating (2-18) (2-19) (2-20) and (2-21') with respect to each parameter and slightly arranging them, we have:

effects of k:

$$\frac{du}{dk} = -\frac{N + (2\pi - 4)\int_0^{L_B} \frac{t}{q} dt + 8\int_0^{\frac{m}{2}} \frac{t}{q} dt + 2\int_0^{L_B} \frac{t}{m}\frac{t}{q} dt}{(2\pi - 4)\int_0^{L_B} \frac{1}{u_x q} dt + 8\int_0^{\frac{m}{2}} \frac{1}{u_x q} dt + 2\int_0^{L_B} \frac{t}{m}\frac{1}{u_x q} dt} < 0. \tag{2-57}$$

$$\frac{dH_B}{dk} = \frac{dL_B}{dk} \gtrless 0. \tag{2-58}$$

$$\frac{dE}{dk}=\frac{-1}{1-\theta}\frac{dL_B}{dk}\gtrless 0. \tag{2-59}$$

effects of y:

$$\frac{du}{dy}=\frac{(2\pi-4)\int_0^{L_B}\frac{1}{q}dt+8\int_0^{\frac{m}{2}}\frac{1}{q}dt+2\int_0^{L_B}\frac{t}{m}\frac{1}{q}dt}{(2\pi-4)\int_0^{L_B}\frac{1}{u_x q}dt+8\int_0^{\frac{m}{2}}\frac{1}{u_x q}dt+2\int_0^{L_B}\frac{t}{m}\frac{1}{u_x q}dt}>0. \tag{2-60}$$

$$\frac{dH_B}{dy}=\frac{dL_B}{dy}=\frac{\left[(2\pi-4)\int_0^{L_B}\frac{1}{q}\left(\frac{1}{u_x}-\frac{1}{u_x(L_B)}\right)dt+8\int_0^{\frac{m}{2}}\frac{1}{q}\left(\frac{1}{u_x}-\frac{1}{u_x(L_B)}\right)dt+2\int_0^{L_B}\frac{t}{m}\frac{1}{q}\left(\frac{1}{u_x}-\frac{1}{u_x(L_B)}\right)dt\right]}{(2\pi-4)\int_0^{L_B}\frac{k}{u_x q}dt+8\int_0^{\frac{m}{2}}\frac{k}{u_x q}dt+2\int_0^{L_B}\frac{t}{m}\frac{k}{u_x q}dt}>0. \tag{2-61}$$

$$\frac{dE}{dy}=-\frac{\left[(2\pi-4)\int_0^{L_B}\frac{1}{q}\left(\frac{1}{u_x}-\frac{1}{u_x(L_B)}\right)dt+8\int_0^{\frac{m}{2}}\frac{1}{q}\left(\frac{1}{u_x}-\frac{1}{u_x(L_B)}\right)dt+2\int_0^{L_B}\frac{t}{m}\frac{1}{q}\left(\frac{1}{u_x}-\frac{1}{u_x(L_B)}\right)dt\right]}{(1-\theta)\left[(2\pi-4)\int_0^{L_B}\frac{k}{u_x q}dt+8\int_0^{\frac{m}{2}}\frac{k}{u_x q}dt+2\int_0^{L_B}\frac{t}{m}\frac{k}{u_x q}dt\right]}<0. \tag{2-62}$$

effects of N:

$$\frac{du}{dN}=-\frac{k}{(2\pi-4)\int_0^{L_B}\frac{1}{u_x q}dt+8\int_0^{\frac{m}{2}}\frac{1}{u_x q}dt+2\int_0^{L_B}\frac{t}{m}\frac{1}{u_x q}dt}<0. \tag{2-63}$$

$$\frac{dH_B}{dN}=\frac{dL_B}{dN}=\frac{1}{(2\pi-4)\int_0^{L_B}\frac{u_x(L_B)}{u_x q}dt+8\int_0^{\frac{m}{2}}\frac{u_x(L_B)}{u_x q}dt+2\int_0^{L_B}\frac{t}{m}\frac{u_x(L_B)}{u_x q}dt}>0. \tag{2-64}$$

$$\frac{dE}{dN}=-\frac{1}{(1-\theta)\left[(2\pi-4)\int_0^{L_B}\frac{u_x(L_B)}{u_x q}dt+8\int_0^{\frac{m}{2}}\frac{u_x(L_B)}{u_x q}dt+2\int_0^{L_B}\frac{t}{m}\frac{u_x(L_B)}{u_x q}dt\right]}<0. \tag{2-65}$$

effects of S:

$$\frac{du}{dS}=-\frac{(2\pi-4)L_B+4m+\frac{L_B{}^2}{m}}{(2\pi-4)\int_0^{L_B}\frac{1}{u_x q}dt+8\int_0^{\frac{m}{2}}\frac{1}{u_x q}dt+2\int_0^{L_B}\frac{t}{m}\frac{1}{u_x q}dt}<0. \tag{2-66}$$

$$\frac{dH_B}{dS}=\frac{dL_B}{dS}=-\frac{\left[(2\pi-4)\int_0^{L_B}\left(\frac{u_x(L_B)q(L_B)}{u_x q}-1\right)dt+8\int_0^{\frac{m}{2}}\left(\frac{u_x(L_B)q(L_B)}{u_x q}-1\right)dt+2\int_0^{L_B}\frac{t}{m}\left(\frac{u_x(L_B)q(L_B)}{u_x q}-1\right)dt\right]}{(2\pi-4)\int_0^{L_B}\frac{u_x(L_B)k}{u_x q}dt+8\int_0^{\frac{m}{2}}\frac{u_x(L_B)k}{u_x q}dt+2\int_0^{L_B}\frac{t}{m}\frac{u_x(L_B)k}{u_x q}dt}<0. \tag{2-67}$$

$$\frac{dE}{dS}=\frac{\left[(2\pi-4)\int_0^{L_B}\left(\frac{u_x(L_B)q(L_B)}{u_x q}-1\right)dt+8\int_0^{\frac{m}{2}}\left(\frac{u_x(L_B)q(L_B)}{u_x q}-1\right)dt+2\int_0^{L_B}\frac{t}{m}\left(\frac{u_x(L_B)q(L_B)}{u_x q}-1\right)dt\right]}{(1-\theta)\left[(2\pi-4)\int_0^{L_B}\frac{u_x(L_B)k}{u_x q}dt+8\int_0^{\frac{m}{2}}\frac{u_x(L_B)k}{u_x q}dt+2\int_0^{L_B}\frac{t}{m}\frac{u_x(L_B)k}{u_x q}dt\right]}>0. \tag{2-68}$$

effects of m:

$$\frac{du}{dm}=\frac{4\int_{\frac{m}{2}}^{L_B}\frac{k}{q}dt-\int_0^{L_B}\frac{t^2}{m^2}\frac{k}{q}dt}{(2\pi-4)\int_0^{L_B}\frac{1}{u_x q}dt+8\int_0^{\frac{m}{2}}\frac{1}{u_x q}dt+2\int_0^{L_B}\frac{t}{m}\frac{1}{u_x q}dt}\gtrless 0. \tag{2-69}$$

$$\frac{dL_B}{dm}=-\frac{1}{u_x(L_B)k}\frac{du}{dm}\gtrless 0. \tag{2-70}$$

$$\frac{dE}{dm}=\frac{1-\frac{dL_B}{dm}}{1-\theta}\gtrless 0 \qquad [\theta_1,\theta_2]. \tag{2-71}$$

$$\frac{dH_B}{dm}=\frac{dL_B}{dm}+(1-\theta)\gtrless 0 \qquad \theta\in[0,\theta_2]. \tag{2-72}$$

effects on r and R:

$$\frac{\partial r(0)}{\partial k}>0,\quad \frac{\partial r(0)}{\partial y}<0,\quad \frac{\partial r}{\partial N}>0,\quad \frac{\partial r}{\partial S}>0 \quad\text{and}\quad \frac{\partial r}{\partial m}=\frac{\partial r}{\partial u}\frac{du}{dm}=-\frac{1}{u_x q}\frac{du}{dm}\gtrless 0,$$

$$\frac{\partial R_{(t=m,\theta=0)}}{\partial k}>0,\quad \frac{\partial R_{(t=m,\theta=0)}}{\partial y}<0,\quad \frac{\partial R}{\partial N}>0,\quad \frac{\partial R}{\partial S}>0 \text{ and}$$

$$\frac{\partial R}{\partial m}=\begin{cases}-\frac{1}{u_X Q}\frac{du}{dm}-\frac{k}{Q}\gtrless 0, & t<m\\ -\frac{1}{u_X Q}\frac{du}{dm}+\frac{k(1-\theta)}{Q}\gtrless 0, & t>m\end{cases}. \tag{2-73}$$

The sign of effects $\frac{dL_B}{dk}$ and $\frac{dH_B}{dk}$ are also determined as negative by using the method as in case A. Moreover, the result, $\frac{dE}{dk}>0$ can be obtained.

Next, we will derive the effect of subcenter location. Differentiating (2-8), (2-16) and (2-17) with respect to m, and rearranging them slightly, we have:

$$\frac{dv}{dm}=\frac{1}{1-\theta}>0 \qquad \theta\in[0,\theta_1]. \tag{2-74}$$

$$\frac{d\theta_1}{dm}=\frac{\frac{dL_B}{dm}m-L_B}{(L_B)^2}. \tag{2-75}$$

$$\frac{d\theta_2}{dm}=\frac{\frac{dL_B}{dm}m-L_B}{(m)^2}. \tag{2-76}$$

First suppose that $\frac{dL_B}{dm}>0$, and then we have $\frac{du}{dm}<0$ in (2-70). Under this assumption, it holds that:

$$\frac{dr}{dm}=-\frac{1}{u_x q}\frac{du}{dm}>0,\quad \frac{dq}{dm}=\frac{dq}{du}\frac{du}{dm}<0 \text{ and}$$

$$\frac{dN_{CBD}}{dm}=[(2\pi-4)L_B+2m]\frac{dL_B}{dm}+2L_B-m>0 \tag{2-77}$$

meaning that if CBD-urban fringe increases with subcenter location, bid rent in CBD area increases everywhere and thereby lot size in the CBD area decreases everywhere and as a result, population in the CBD area must increase. How will the SBD area change with subcenter location? In this circumstance, from (2-75) and (2-76), it is found that θ_1 and θ_2 change in the same direction, but direction is not determined. Thus analysis is performed for each direction.

Case 1. We suppose $\frac{dL_B}{dm}m-L_B>0$ thus $\frac{d\theta_1}{dm}>\frac{d\theta_2}{dm}>0$. Denoting the area of SBD area by Z, it is found that $\frac{dZ}{dm}=[2L_B-m]+2m\frac{dL_B}{dm}+\frac{(L_B)^2}{3m^2}\left[3m\frac{dL_B}{dm}-L_B\right]>0$. We demonstrate that the SBD area expands in each direction in case 1. In Figure 2-6, the thick-solid and thick-dotted lines respectively express the urban outline before and after the subcenter location change from A to A', and, BD line and CF line respectively express the demarcation line before and after subcenter location change. θ_1 and θ_2 increase respectively to θ_1' and θ_2', and we draw a straight line from CBD via point D to point E, the intersection with CF. From (2-74), we see that the distance from A' to the point on line CE is larger than the distance from A to the point on line BD for each $\theta \in [0, \theta_1]$. Moreover, we see that the distance from A' to any point on line EF is greater than OE, and thus greater than OD (the previous urban fringe). This implies that the distance from A' to any point on line EF is larger than the distance from A to any point on line DG. It is obvious that the distance from A' to the new SBD-urban fringe is larger than the distance from A to the original SBD-urban fringe since L_B is increased. Putting it another way, if we clip the previous SBD area from the paper and paste it to the new SBD area, such that point A is located to A', it is found that the previous SBD area is completely included in the new SBD area.

In case 2, it is supposed that $\frac{dL_B}{dm}m - L_B < 0$, and thus $\frac{d\theta_1}{dm} < \frac{d\theta_2}{dm} < 0$. By referring to Figure 2-7 and by similar procedure we used in case 1, we can easily demonstrate that SBD area expands in every direction in this case as well. We thus conclude that when $\frac{dL_B}{dm} > 0$ and thus $\frac{du}{dm} < 0$, SBD area must expand in every direction. By the same logic as in case A, this implies that population in both CBD area and SBD area must increase, which contradicts the fact that total population in the city is constant. Therefore, it can not be held that $\frac{dL_B}{dm} > 0$, and it must be that $\frac{dL_B}{dm} < 0$. To sum up, the following are derived.

$$\frac{dL_B}{dm} \le 0 \text{ and } \frac{dL_B}{dm} \ge -1. \tag{2-78}$$

$$\frac{du}{dm} \ge 0. \tag{2-79}$$

$$\frac{dr}{dm} \le 0. \tag{2-80}$$

$$\frac{dH_B}{dm} = \begin{Bmatrix} 1+\dfrac{dL_B}{dm} \geq 0, & \theta = 0. \\ \dfrac{dL_B}{dm} \leq 0, & \theta = 1. \end{Bmatrix}. \tag{2-81}$$

$$\frac{d\theta_1}{dm} = \frac{\dfrac{dL_B}{dm}m - L_B}{\left(L_B\right)^2} \leq 0 . \tag{2-82}$$

$$\frac{d\theta_2}{dm} = \frac{\dfrac{dL_B}{dm}m - L_B}{m^2} \leq 0 . \tag{2-83}$$

$$\frac{dE}{dm} = \frac{1-\dfrac{dL_B}{dm}}{1-\theta} \geq 0 . \tag{2-84}$$

$$\frac{dR}{dm} = \begin{Bmatrix} -\dfrac{1}{u_X Q}\dfrac{du}{dm} - \dfrac{k}{Q} \leq 0, & t < m. \\ -\dfrac{1}{u_X Q}\dfrac{du}{dm} + \dfrac{k(1-\theta)}{Q} \gtrless 0, & t > m. \end{Bmatrix}. \tag{2-85}$$

The results concerning case B are summarized in Table 2-2.

2-4 Concluding Remarks

In this paper, we built a closed city model with a subcenter. The urban shape for each case was discussed in detail and comparative static analysis was performed. In this final section, we summarize the main results.

As shown in both Tables 2-1 and 2-2, an increase in commuting cost decreases the utility level, contracts city size, and raises rent at two centers.

Raising income level necessarily increases the welfare level and city size. Land rents of both centers are decreased. A larger population expands city size and lowers welfare. The bid rents of all locations are raised by larger population in both cases.

As for the effect of rural land price, we find that higher rural land price contracts the city size, lowers welfare and increases bid rent in each location[2].

Reviewing Wheaton's [1974] and Sasaki's [1987] paper, it is found that establishment of a subcenter does not change the effect of parameters in the basic monocentric urban model. An intuitive interpretation to this result is as follows. As shown in Figures, most parts, at least half of CBD area still takes a circular form in equilibrium even after a subcenter is established. Thus, people in that circular part will react to a change in parameters in a way similar to that in a monocentric circular city case; e.g., $\frac{du}{dk} < 0$. Since residents in a city are identical, people residing in non-circular, SBD area will, in equilibrium, react in the same way as those in CBD area. Thus, comparative statics results in the present model do not differ from the ones in a monocentric model [see Wheaton [1974], David Pines & Efraim Sadka [1986], Brueckner [1987] and Sasaki [1987]].

The effect of subcenter location is displayed in the last row in both Tables 3.1 and 3.2. It is implied that utility level increases as subcenter location becomes farther from a CBD. Bid rent of residents in CBD area and bid rent of residents with distance from CBD less than subcenter location in SBD area are decreased by moving the subcenter outward, while the bid rent of those with distance from CBD greater than subcenter location possibly increases or decreases with a new worksite, because that effect includes two opposing items: from the second part of (56) and (73), it is seen that the first item necessarily decreases due to utility level's increase while the second one increases because of saved commuting cost (since to residents living in area where $t > m$, commuting distance is shorter than before). Therefore if the first item is less/greater than the second one, for a resident located further from the CBD than the subcenter, bid rent will increase/decrease with subcenter location[3].

As described above, the unambiguous results of comparative static analysis in the paper are due to some restrictive assumptions. In particular, our model has neglected the behavior of a firm

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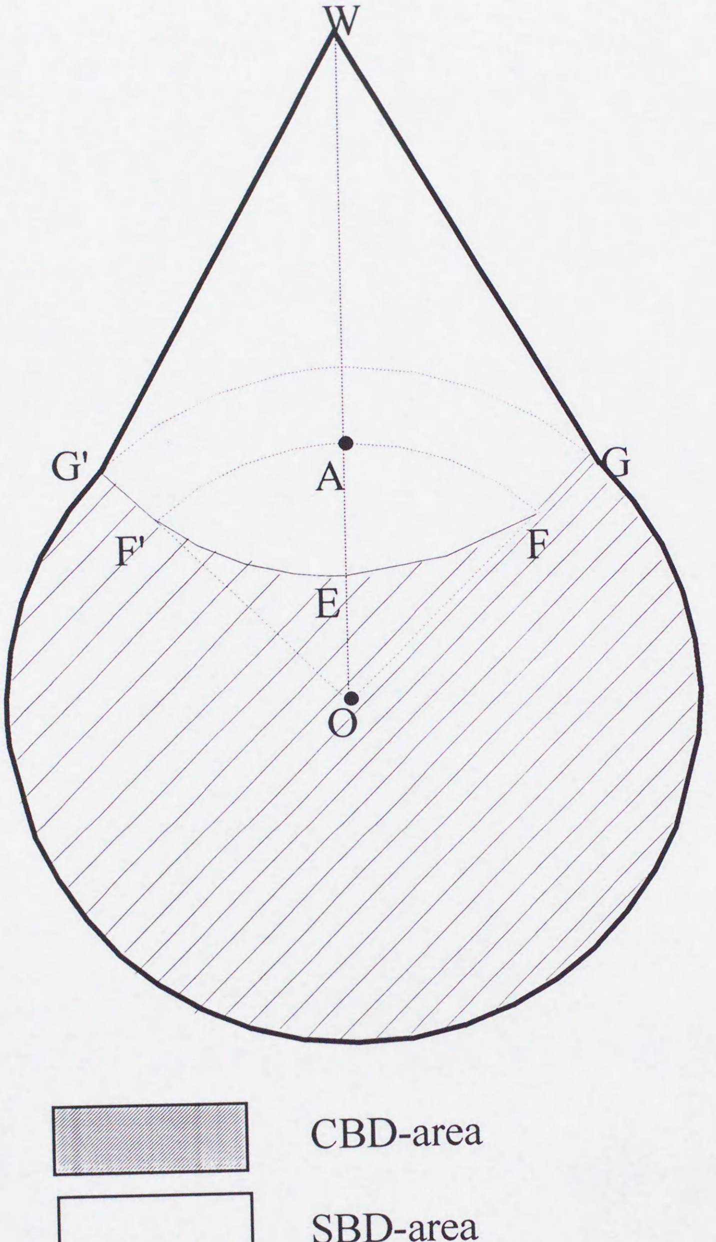
2 One unambiguous result in the Ross and Yinger's(1995) open city setting is the effect of rural land price. It is also shown that the city size contracts as rural land price increases.

3 If the total land rent in a city decreases as subcenter location becomes farther from a CBD, then the welfare of absentee land lords decreases while that of residents in a city increases.

and implicitly assumed that firms adjusted their locations in CBD or SBD so as to employ all people who want to work in each center, and offer the same wage rate in two centers. This implies that no agglomeration economies exists in industry. For instance, if communication among firms is necessary to increase the production efficiency and its communication cost changes in proportion to the distance between firms, the farther distance between CBD and SBD will cause higher cost or lower efficiency to firms which move to SBD, and thereby will lower the wage rate at SBD. Under such circumstance with agglomeration economies, the effect of subcenter location on the welfare of residents is complicated. To repeat, recognizing limitations of a model, this paper was intended to derive unambiguous results of comparative static analysis in a two-center city by employing an Alonso-type residential land use model where the locations of two export nodes and thus locations of industrial activity are historically or institutionally given.



EF or EF': Inner-demarcation line
FG or F'G': Outer-demarcation line
Arc GG'(solid line): CBD-urban fringe
WG or WG': SBD-urban fringe

Figure 2-1. Urban shape of a duo-centric city

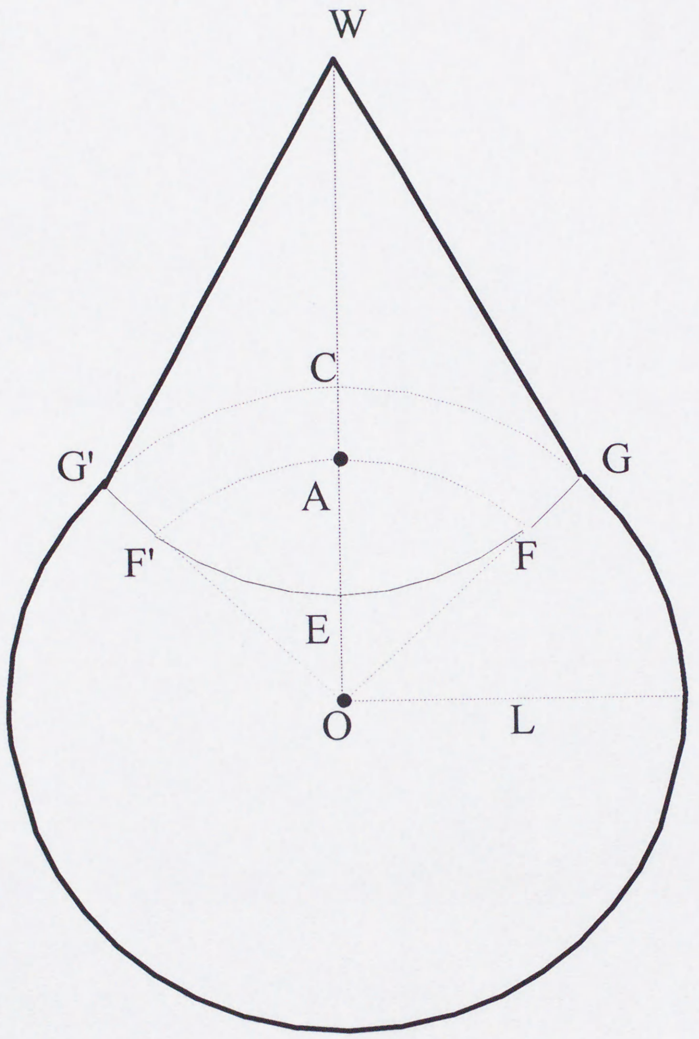


Figure 2-2. Urban shape of case A for a duo-centric city

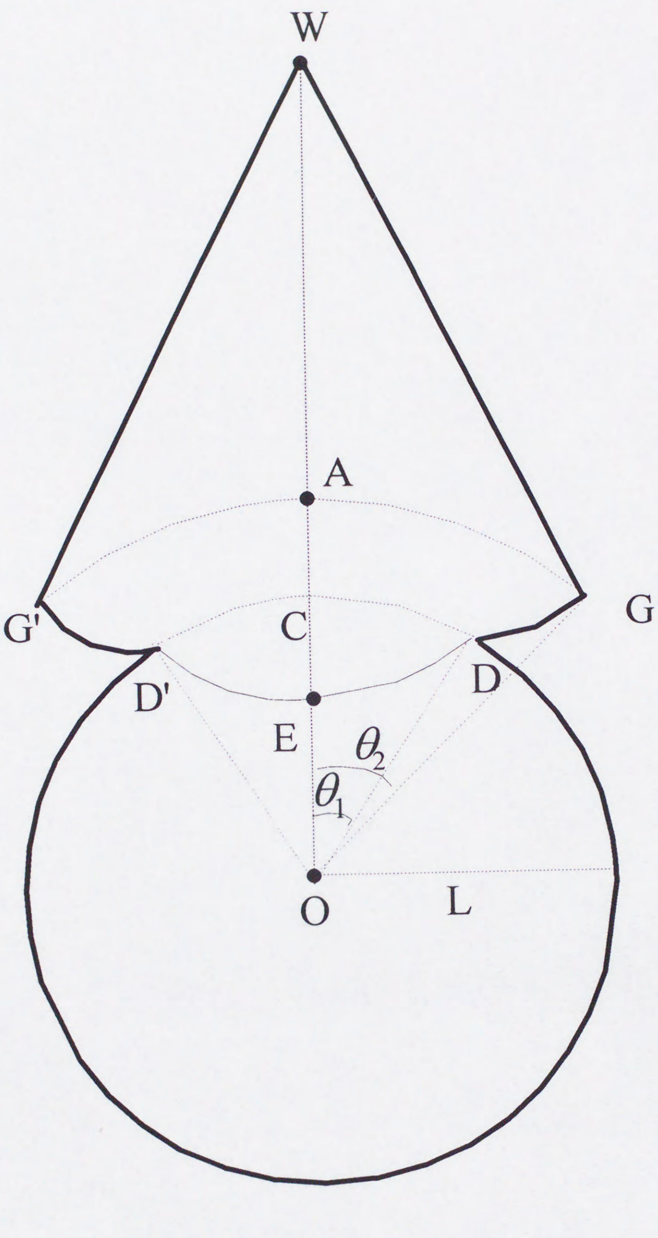


Figure 2-3. Urban shape of case B for a duo-centric city

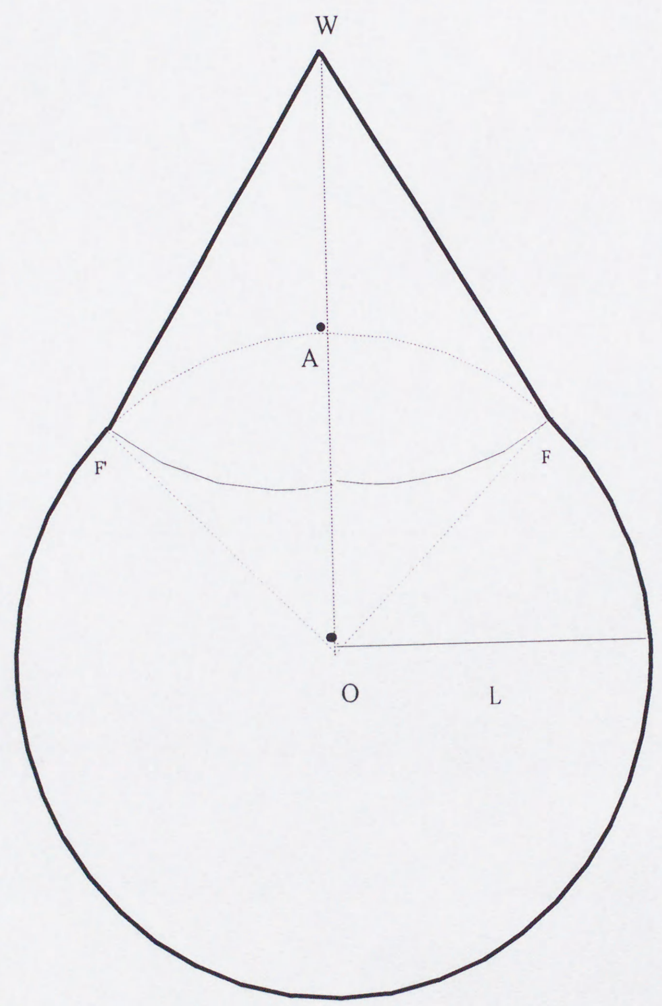


Figure 2-4 Urban shape of closed city where CBD-urban fringe is equal to subcenter location

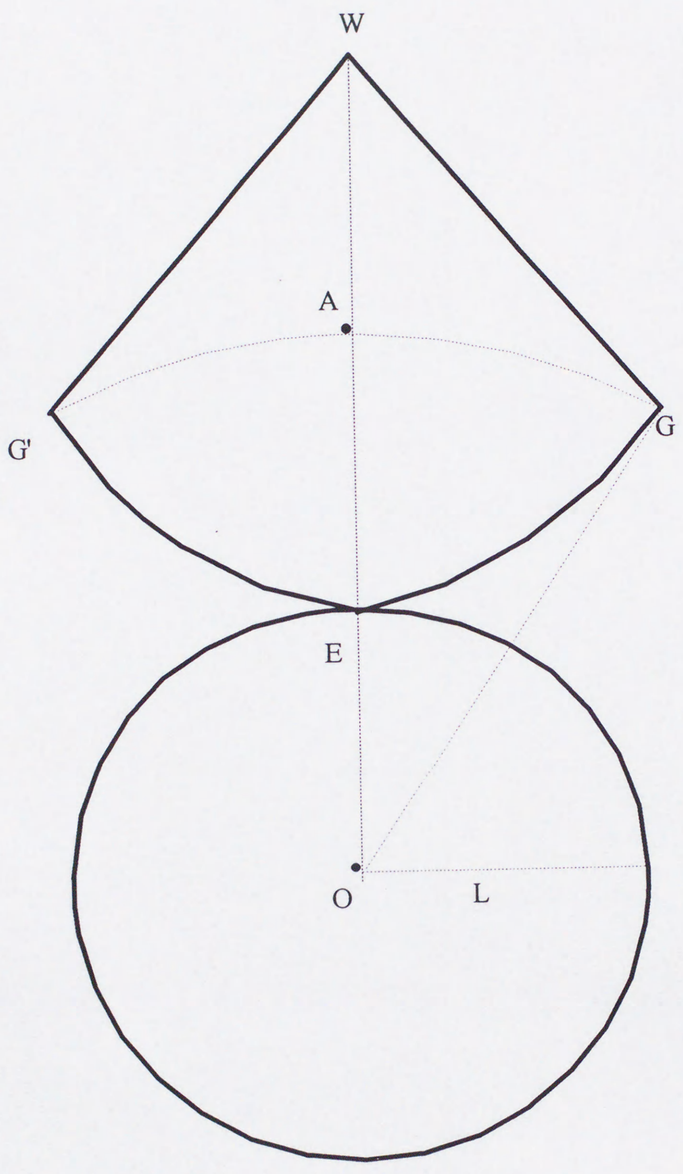


Figure 2-5 Urban shape of a closed city where subcenter is created on its maximum subcenter location

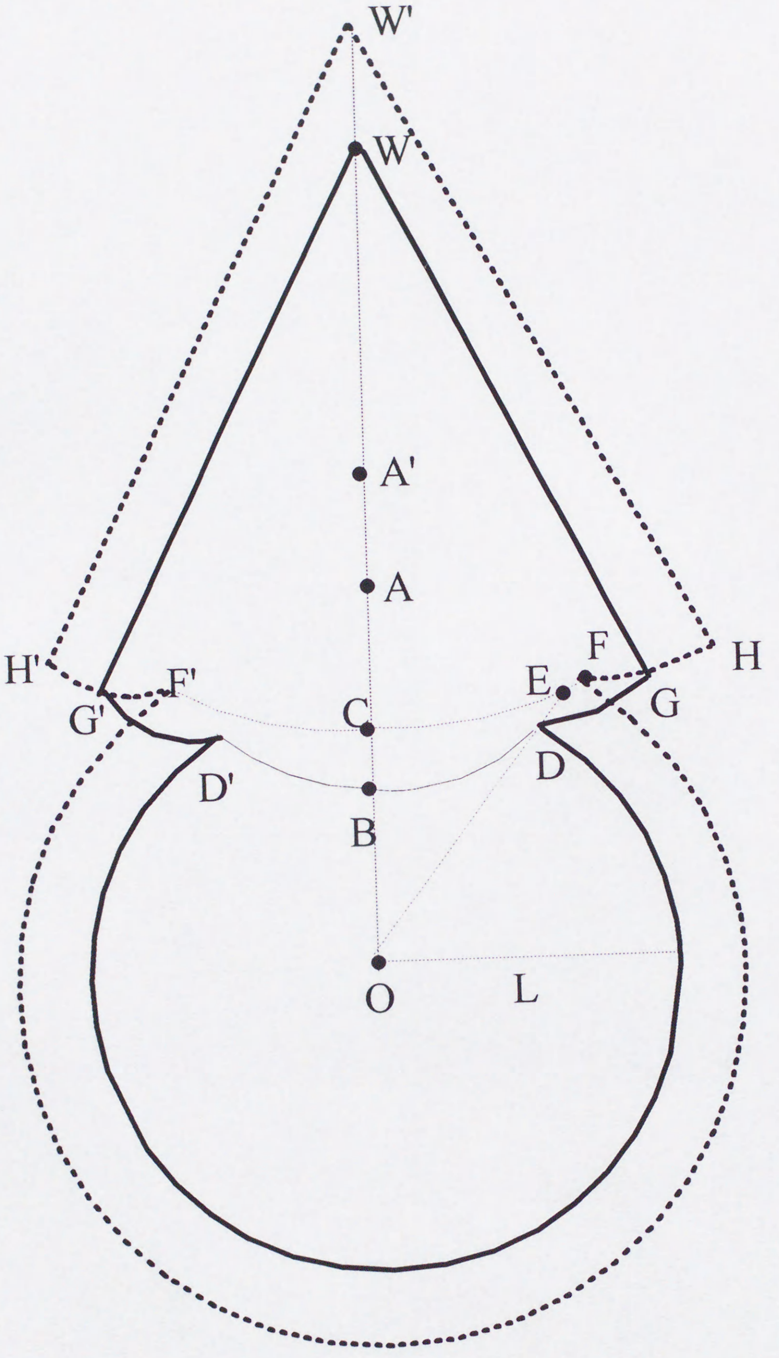


Figure 2-6. Urban shape before and after SBD location change: case 1

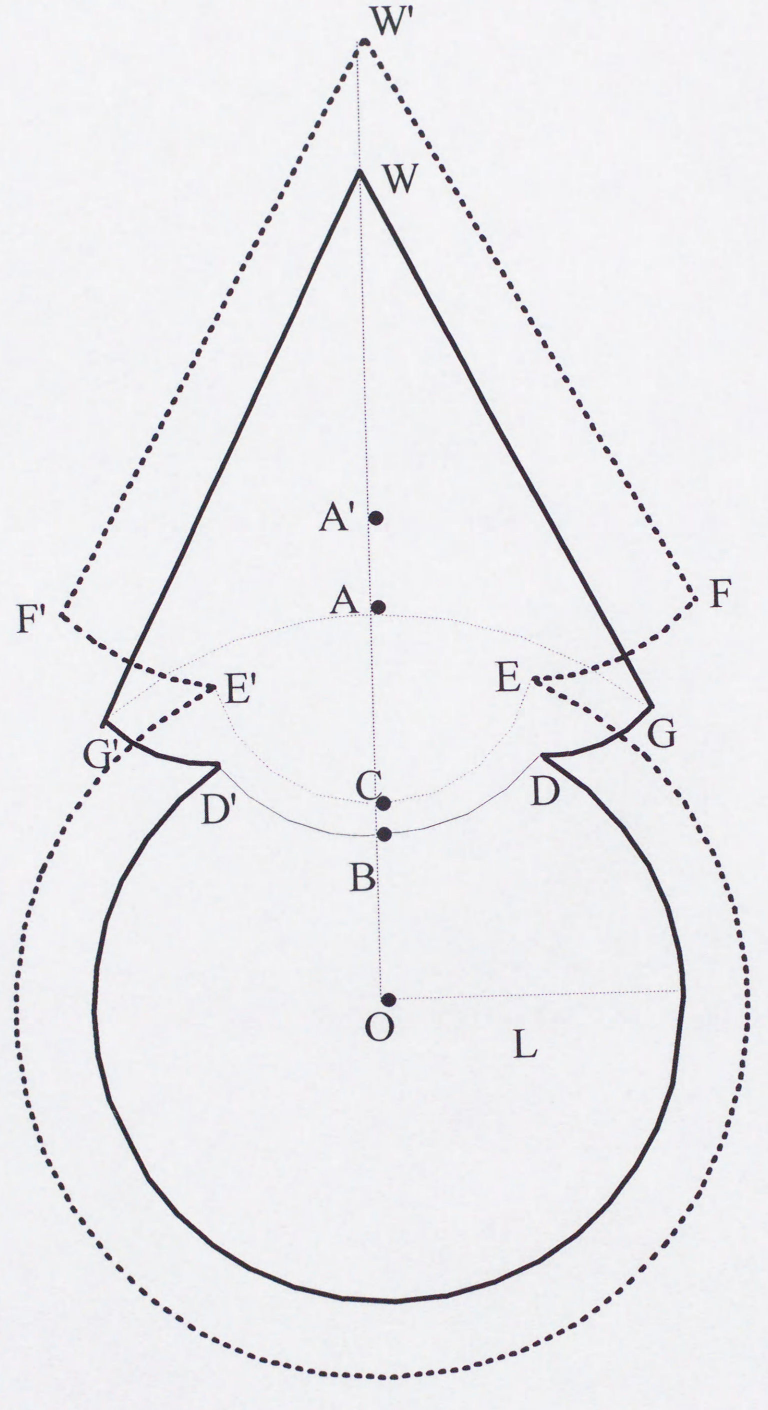


Figure 2-7. Urban shape before and after SBD location change: case 2

Table 2-1. Results of comparative static analysis of case A for a closed city with a subcenter

Effect on:	u	L_A	H_A	r	R
Increase in					
k	$-$	$-$	$-$	$+(t=0)$	$+(t=m, \theta=0)$
y	$+$	$+$	$+$	$-(t=0)$	$-(t=m, \theta=0)$
N	$-$	$+$	$+$	$+$	$+$
S	$-$	$-$	$-$	$+$	$+$
m	$+$	$-$	$+(\theta=0)$ $-(\theta=1)$	$-$	$-(t<m)$, $?(t>m)$

Table 2-2. Results of comparative static analysis of case B for a closed city with a subcenter

Effect on:	u	L_B	E	H_B	r	R
Increase in						
k	−	−	+	−	$+ (t = 0)$	$+ (t = m, \theta = 0)$
y	+	+	−	+	$- (t = 0)$	$- (t = m, \theta = 0)$
N	−	+	−	+	+	+
S	−	−	+	−	+	+
m	+	−	+	$+ (\theta = 0)$ $- (\theta = 1)$	−	$-(t < m)$, $?(t > m)$

Appendix

As for the derivation of (2-15'), by multiplying $-k$ on both sides of equation (2-15) and using (2-5), (2-11) and principal of integration by parts, we have:

$$(2\pi - 2)\left(SL - \int_0^L r dt\right) - 2\int_0^{\frac{m}{2}} r dt + 2\int_{\frac{m}{2}}^{m} r dt + 2\int_{\frac{m}{2}}^{m} R\left(2 - \frac{m}{t}, t\right) dt - 2\int_{\frac{m}{2}}^{m} R(0,t) dt$$

$$+2\int_m^L \frac{t}{m} R(1,t) dt + 2\int_L^{L+m} \frac{t}{m} R\left(1 - \frac{t-L}{m}, t\right) dt - 2\int_m^{m+L} \frac{t}{m} R(0,t) dt = -kN \qquad \text{(A-1)}$$

The fourth term on the left side of (A-1) is rewritten as:

$$2\int_{\frac{m}{2}}^{m} R\left(2 - \frac{m}{t}, t\right) dt$$

$$= 2\int_{\frac{m}{2}}^{m} \frac{y - X - k\left[m - t + t(2 - \frac{m}{t})\right]}{Q} dt$$

$$= 2\int_{\frac{m}{2}}^{m} \frac{y - X - kt}{Q} dt \qquad \text{(A-2)}$$

Since households will pay the same rent as long as they occur the same commuting cost, then (A-2) can be rewritten as:

$$= 2\int_{\frac{m}{2}}^{m} \frac{y - x - kt}{q} dt$$

$$= 2\int_{\frac{m}{2}}^{m} r dt \qquad \text{(A-3)}$$

Similarly, the last four parts on the left side of (A-1) will be respectively written:

$$-2\int_{\frac{m}{2}}^{m} R(0,t)dt$$

$$= -2\int_{0}^{\frac{m}{2}} rdt \qquad \text{(A-4)}$$

$$+2\int_{m}^{L} \frac{t}{m} R(1,t)dt$$

$$= 2\int_{m}^{L} \frac{t}{m} rdt \qquad \text{(A-5)}$$

$$+2\int_{L}^{m+L} \frac{t}{m} R\left(1-\frac{t-L}{m}, t\right)dt$$

$$= Sm + 2SL \qquad \text{(A-6)}$$

and

$$-2\int_{m}^{m+L} \frac{t}{m} R(0,t)dt$$

$$= 2\int_{0}^{L} \frac{m+t}{m} rdt \qquad \text{(A-7)}$$

Substituting (A-3)-(A-7) into (A-1) and slightly arranging it, we have:

$$2\pi\left(SL_A - \int_0^{L_A} rdt\right) - 4\int_0^{\frac{m}{2}} rdt + 4\int_{\frac{m}{2}}^{m} rdt + Sm - 2\int_0^{m} \frac{t}{m} rdt = -kN \qquad \text{(A-8)}$$

Population conditions of other cases are obtained in the same way, thus the detailed processes for other cases are omitted here.

Chapter 3

Model of an open city with a subcenter

3-1 Introduction

In an open city model, the utility of residents equals that of the rest of the economy, which is exogenously fixed, while the population of the city is determined endogenously. The open city model is usually used to describe urban conditions in developing countries that have surplus labor in rural areas. Due to a large increase in immigrant coming from rural areas, metropolitan population steeply increases in many developing countries so as to cause decentralization. A study on duo-centric open city is pioneered by Yinger [1992]. In his model, two groups of residents who hold different income level respectively locate in CBD and SBD zones. Transportation system in the city includes not only an infinite radial way radiating from CBD, but dense circular rings running around the CBD. Four urban spatial configurations and urban shapes are discussed in detail, and boundary conditions of two residential zones for each case are obtained. The paper shed the light on how the second employment center influences urban residential structure and how increases in suburban income alter the shape of the city and suburban residential zones. However, the impact of other index such as transportation cost, utility level and rural land price are not evaluated in his work.

Recently Ross and Yinger [1995] performed a comparative static analysis in the setting of an open, two-center city. Their model is general in that wage rates are endogenously determined and land input to production is considered. However, in return for the 'generality' of a model, the sign is ambiguous in many cases: only the effect of a change in agriculture land rent is clearly determined. The effect of transportation cost change is a great concern to urban-policy makers, but its effect on population size, for instance, is ambiguous. Such 'indeterminancy' in a two-center city model is predictable. This is because even in a monocentric-model, whether a closed or an open city model, the comparative static results are ambiguous in many cases. While Ross and Yingers' paper treats an open city, Sasaki and Kaiyama [1990] deal with a closed city. Focusing on the effects of transportation cost, the followings are only clear results: $\frac{du}{dk_c} < 0$, $\frac{du}{dk_p} < 0$ and $\frac{db}{dk_c} < 0$ in a closed city, where u, k_c and k_p are, respectively, utility level of a resident, freight transport cost and commuting transport cost. Sasaki [1987b]'s model is much simpler in that wage rate is exogenously given and freight transport cost is not considered, but even within that framework the effects of k_p on the boundary between industrial and residential districts, and income level are indeterminate. This suggests that treating the wage rate as exogenous will not work much so as to reduce or eliminate the ambiguity of comparative statics, but land consumption behavior of industry is crucial to obtain unambiguous results.

In the more recent research of Zhang and Sasaki [1997], they carry out comparative static analysis in the setting of a closed city with a subcenter where land is not used by firms and wage rate is exogenous, obtaining unambiguous results in almost all of cases including the impacts on the land rents. Needless to say, it is restrictive to assume that firms do not demand land as an input into production, and thereby competition in the land market between firms and households does not occur. Recognizing this limitation, the present paper intends to analyze the effects of subcenter formation, as a first approximation, using an open city model where no land is consumed by firms. In particular, the effects of a subcenter location will be analyzed as well, which were not studied in Ross and Yinger [1995].

The model described in the second section is along the line of conventional open urban general equilibrium model (Alonso[1964]-type), but incorporates Anas and Moses [1979]'s transportation system. Note that the model in this section assume that residential income does not include land rents, an assumption usually associated with absentee landlords. Because the disappearance of rents seems unrealistic, an open model in which rents are distributed equally among all city residents has been developed (Sasaki [1987a] and Fujita [1989 P. 61]). However, as shown in Fujita [1989, P. 62], in an open model, the pattern of land use is essentially the same whether one assumes land ownership is private or public. The model presented here does not analyze the effects of changes in non-labor income. Comparative static analysis is developed in section 3 employing Sasaki's [1987a] approach. Some main conclusions are summarized in the last section.

3-2 Model

3-2-1 Model of monocentric open city

In this section, we use the approach stated in chapter 3 to create an open model. The behavior of consumers is as well as expressed as in (2-1) and (2-2). Urban transportation system includes not only an infinite number of spoke-roads coming out from the CBD, but an infinite number of circle streets running around the CBD. The equilibrium conditions of such open city are shown in (2-6) and (2-7) or ((2-7')), in which population and urban fringe are separately determined.

3-2-2 Model of an open city with a subcenter

Now we exogenously give a subcenter location (see point A in Figure 3-1) where the city government wants to establish. Location behavior of firms is not considered in this model, and it is just assumed that the location A becomes a new employment center (subcenter) and a sufficient number of firms locate there so as to employ all the people who want to work there. The distance between two centers is m. Moreover it is assumed that firms in the subcenter offer the same wage level as those in the CBD. Following Zhang and Sasaki [1995], we denote the area occupied by residents who commute to the CBD and the SBD, respectively, by the CBD-area and the SBD-area. According to the assumed transportation system, to a resident in CBD-area, the most efficient route for commuting to worksite is a spoke road connecting his residence and CBD, while to a resident in SBD-area, his most efficient commuting route is different depending on the locations of his residence and subcenter: in terms of the distance to the CBD, if his location is closer than the subcenter location, he should reach the spoke through the subcenter by the circle street via his location first, then goes to the SBD by that spoke; if his location is further than the subcenter location, he will get to the circle street through the subcenter by the spoke via his location, and then reach the SBD by that circle street. Between the two residential zones, there must be a demarcation line, on which consumers incur the same commuting cost whether they work in the CBD or SBD. We define the section on demarcation line closer than subcenter as inner-demarcation, and its mathematical expression has been presented in (2-8) (see also Zhang and Sasaki [1997] for detailed procedure).

Since the utility level is given exogenously, CBD-urban fringe, i.e., the fringe of CBD-area should be the same as the urban fringe of previous monocentric city, i.e., b. Letting H denote the SBD-urban fringe that is represented by line WG or WG' in Figure 3-1, it will be determined by the following formula:

$$H(\theta) = b + (1-\theta)m \qquad \theta \in [0,1]. \tag{3-5}$$

Depending on the location of new worksite, that is, depending on whether the CBD-urban fringe b is less or greater than subcenter location m, the new urban shape will be different. Thus like Zhang and Sasaki [1997], we have to discuss each of them.

Case A.

In case A, it is that $b > m$. A typical urban shape in this case is shown in Figure 3-1. Since the utility level is constant in an open city, the CBD-urban fringe is always equal to b regardless of the subcenter location, and thus case A implies that the subcenter is built inside the monocentric city. The SBD-urban fringe H_A and population N can be respectively obtained by the following system:

$$H_A(\theta) = b + (1-\theta)m \qquad \theta \in [0,1]. \tag{3-6}$$

$$\begin{aligned}(2\pi-2)\int_0^b \frac{t}{q}dt + 2\int_0^{\frac{m}{2}}\int_0^1 \frac{t}{q}d\theta dt + 2\int_{\frac{m}{2}}^{m}\int_{2-\frac{m}{t}}^{1} \frac{t}{q}d\theta dt + 2\int_{\frac{m}{2}}^{m}\int_0^{2-\frac{m}{t}} \frac{t}{Q}d\theta dt \\ +2\int_m^b\int_0^1 \frac{t}{Q}d\theta dt + 2\int_b^{m+b}\int_0^{1-\frac{t-b}{m}} \frac{t}{Q}d\theta dt = N\end{aligned} \tag{3-7}$$

or

$$2\pi\left(Sb - \int_0^b r dt\right) - 4\int_0^{\frac{m}{2}} r dt + 4\int_{\frac{m}{2}}^{m} r dt + Sm - 2\int_0^m \frac{t}{m} r dt = -kN. \tag{3-7'}$$

Equation (3-6) expresses the relation between two urban fringes. Relation (3-7) or its alternative form (3-7') represents the population accommodation condition of new urban structure. In (3-7), the first three terms on the left-hand side denote the population in the CBD area, the fourth part and the sum of fifth and sixth parts respectively expresses the number of people whose location is closer and further than subcenter location in the SBD area. Derivation of (3-7') is shown in Appendix of chapter 3.

Case B.

In case B, it is that $b < m$. Putting it another way, if a subcenter is established at some point outside the monocentric city, case B will emerge. The urban shape in this case is somewhat complicated, which is shown in Figure 3-2. We determine first the urban shape and the boundary between residential areas. In Figure 3-2, since b is less than m, the arc of the CBD-urban fringe must intersect with the inner-demarcation line at some point (e.g., at point D or D'), and we can draw an Iso-bid rent curve with changing θ until it intersects with the circle street via the SBD at point G or G', on which the bid rent is equal to S. Point W is easily determined by adding b from point A along the vertical line, and we connect W and G or G' by an Iso-bid rent curve on which the rent is equal to agricultural land rent. Then the urban shape will be depicted as the thickest outline in Figure 3-2. In this case, there is no outer-demarcation line, and the SBD-urban fringe includes two parts: we define, respectively, the part closer than subcenter location as inner-SBD-urban fringe E and that further than subcenter location as outer-SBD-urban fringe H_B. Defining $\angle AOD$ in Figure 3-2 as θ_1 and $\angle AOG$ as θ_2, it holds that:

$$\theta_1 = 2 - \frac{m}{b}. \tag{3-8}$$

$$\theta_2 = \frac{b}{m}. \tag{3-9}$$

Two SBD-urban fringes E and H_B, and population N will be obtained by the following system:

$$E(\theta) = \frac{m-b}{1-\theta} \qquad \theta \in [\theta_1, \theta_2]. \tag{3-10}$$

$$H_B(\theta) = b + (1-\theta)m \qquad \theta \in [0, \theta_2]. \tag{3-11}$$

$$2(\pi - \theta_1)\int_0^b \frac{t}{q} dt + 2\int_0^{\frac{m}{2}} \int_0^{\theta_1} \frac{t}{q} d\theta dt + 2\int_{\frac{m}{2}}^{m} \int_0^{\theta_1 - \left(2 - \frac{m}{t}\right)} \frac{t}{q} d\theta dt + 2\int_{\frac{m}{2}}^{b} \int_0^{2-\frac{m}{t}} \frac{t}{Q} d\theta dt$$

$$+2\int_{b}^{m}\int_{0}^{1-\frac{m-b}{t}}\frac{t}{Q}d\theta dt+2\int_{m}^{m+b}\int_{0}^{1-\frac{t-b}{m}}\frac{t}{Q}d\theta dt=N \tag{3-12}$$

or

$$(2\pi-4)\left(Sb-\int_{0}^{b}rdt\right)-8\int_{0}^{\frac{m}{2}}rdt-2\int_{0}^{b}\frac{t}{m}rdt++4Sm+\frac{Sb^{2}}{m}=-kN\,. \tag{3-12'}$$

In population condition (3-12), the first three parts on the left hand side express the CBD area population; the sum of fourth, fifth and sixth parts denotes the number of people at the locations closer than a subcenter location in the SBD area; and the last term shows the population residing at the locations further than a subcenter location in the SBD area.

If a subcenter is built on the monocentric urban fringe, there will emerge a special case between cases A and B. The description for the case is shown in Figure 3-3. In this case, SBD-urban fringe H' and population N will be exogenously obtained by the following system:

$$H'(\theta)=2b-\theta b \qquad \theta\in[0,1]\,. \tag{3-13}$$

$$2(\pi-2)\int_{0}^{b}\frac{t}{q}dt+2\int_{0}^{\frac{b}{2}}\int_{0}^{1}\frac{t}{q}d\theta dt+2\int_{\frac{b}{2}}^{b}\int_{0}^{1-\left(2-\frac{b}{t}\right)}\frac{t}{q}d\theta dt+2\int_{\frac{b}{2}}^{b}\int_{0}^{2-\frac{b}{t}}\frac{t}{Q}d\theta dt$$

$$+2\int_{b}^{2b}\int_{0}^{1-\frac{t-b}{b}}\frac{t}{Q}d\theta dt=N \tag{3-14}$$

or

$$2\pi\left(Sb-\int_{0}^{b}rdt\right)-4\int_{0}^{\frac{b}{2}}rdt+4\int_{\frac{b}{2}}^{b}rdt+Sb-2\int_{0}^{b}\frac{t}{b}rdt=-kN\,. \tag{3-14'}$$

It is easy to see that if a subcenter is established at a location further than $2b$, the initial monocentric city will be divided into two cities, and the subcenter will become a new CBD of a new city (see Figure 3-4). Therefore, there exists the furthest subcenter location in case B, i.e., $m=2b$. If m exceeds $2b$, there will be rural land between the CBD area and the SBD area. Denoting the furthest location and two SBD-urban fringes, respectively, by m^*, E^*, and H^*, it is seen that $m^*=2b$, $\theta_1=0$ and $\theta_2=\frac{1}{2}$, and two SBD-urban fringes E^*, H^* and population N are respectively determined by the following system:

$$E^*(\theta) = \frac{b}{1-\theta} \qquad \theta \in \left[0, \frac{1}{2}\right]. \tag{3-15}$$

$$H^*(\theta) = 3b - 2\theta b \qquad \theta \in \left[0, \frac{1}{2}\right]. \tag{3-16}$$

$$2\pi \int_0^b \frac{t}{q} dt + 2\int_b^{2b} \int_0^{1-\frac{b}{t}} \frac{t}{Q} d\Theta dt + 2\int_{2b}^{3b} \int_0^{\frac{3b-t}{2b}} \frac{t}{Q} d\Theta dt = N \tag{3-17}$$

or

$$2\pi\left(Sb - \int_0^b r dt \right) - 4\int_0^b r dt - 2\int_0^b \frac{t}{b} r dt + \frac{Sb}{2} = -kN . \tag{3-17'}$$

3-3 Comparative Static Analysis

Since the case represented by the system in (3-13) and (3-14) is peculiar case, we do not perform comparative static analysis in that case, but only focus on cases A and B. Because the behavior of consumers stated here is same as that in chapter 3, mathematical relationships of (2-29), (2-30) and (2-31) are also suited in present open model.

3-3-1 Analysis for case A

For the effect of changes in k, the procedure of the comparative static analysis is same as done in chapter 3:

Equation (2-6) is differentiated with respect to k and by using (5) we obtain:

$$\frac{db}{dk} = -\frac{b}{k} < 0 . \tag{3-18}$$

Equation (3-6) is differentiated with respect to k to give:

$$\frac{dH_A}{dk} = \frac{db}{dk} = -\frac{b}{k} < 0 . \tag{3-19}$$

Equation (3-7') is also differentiated with respect to k to give:

$$\frac{dN}{dk}=-\frac{N+2\pi\int_0^b \frac{t}{q}dt+4\int_0^{\frac{m}{2}}\frac{t}{q}dt-4\int_{\frac{m}{2}}^{m}\frac{t}{q}dt+2\int_0^m \frac{t}{m}\frac{t}{q}dt}{k}<0. \tag{3-20}$$

To find the effects of changes in y, the same procedure was applied with relations (2-29)-(2-31):

$$\frac{dH_A}{dy}=\frac{db}{dy}=\frac{1}{k}>0. \tag{3-21}$$

$$\frac{dN}{dy}=\frac{2\pi\int_0^b \frac{1}{q}dt+4\int_0^{\frac{m}{2}}\frac{1}{q}dt-4\int_{\frac{m}{2}}^{m}\frac{1}{q}dt+2\int_0^m \frac{t}{m}\frac{1}{q}dt}{k}>0. \tag{3-22}$$

For the effect of changes in u: Differentiating (2-6), (3-6) and (3-7') with respect to u, and slightly arranging them, we obtain:

$$\frac{dH_A}{du}=\frac{db}{du}=-\frac{1}{u_x(b)k}<0. \tag{3-23}$$

$$\frac{dN}{du}=-\frac{2\pi\int_0^b \frac{1}{u_x q}dt+4\int_0^{\frac{m}{2}}\frac{1}{u_x q}dt-4\int_{\frac{m}{2}}^{m}\frac{1}{u_x q}dt+2\int_0^m \frac{t}{m}\frac{1}{u_x q}dt}{k}<0. \tag{3-24}$$

As for the effect of S, from the three basic equations we obtain:

$$\frac{dH_A}{dS}=\frac{db}{dk}=-\frac{q(b)}{k}<0. \tag{3-25}$$

$$\frac{dN}{dS}=-\frac{2\pi b+m}{k}<0. \tag{3-26}$$

In a similar way, we can analyze the effect of subcenter location m. We have:

$$\frac{db}{dm} = 0. \tag{3-27}$$

$$\frac{dH_A}{dm} = 1 - \theta > 0, \qquad \theta \in [0,1]. \tag{3-28}$$

$$\frac{dN}{dm} \gtrless 0. \tag{3-29}$$

Taking advantage of the results so far, it is possible to derive the effects on the market land rents of both residential areas by using (2-5) and (3-4), i.e.

$$\begin{aligned} &\frac{\partial r}{\partial k} < 0, \quad \frac{\partial r}{\partial y} > 0, \quad \frac{\partial r}{\partial u} < 0, \quad \frac{\partial r}{\partial S} = 0 \text{ and } \frac{\partial r}{\partial m} = 0, \\ &\frac{\partial R}{\partial k} < 0, \quad \frac{\partial R}{\partial y} > 0, \quad \frac{\partial R}{\partial u} < 0, \quad \frac{\partial R}{\partial S} = 0 \text{ and } \frac{\partial R}{\partial m} = \begin{cases} < 0, & t < m \\ > 0, & t > m \end{cases}. \end{aligned} \tag{3-30}$$

Based on the results obtained thus far, we can determine the sign of $\frac{dN}{dm}$ in (3-29) in the following way. Suppose that $\frac{dN}{dm} < 0$. Since utility level, income level, price of composite good and commuting cost per round mile are constant, a decrease in population means that city size must contract in either CBD-area or SBD-area or both areas. Differentiating (3-1) with respect to m, we have:

$$\frac{dv(\theta)}{dm} = \frac{1}{2-\theta} > 0, \qquad \theta \in [0,1]. \tag{3-31}$$

From (3-27) and (3-31), we conclude that CBD-area expands with subcenter location. As for SBD-area, we can also prove it to be expanded with subcenter location by referring to Figure 3-5. In Figure 3-5, suppose that a subcenter moves from A to A'. EE' curve and FF' curve, respectively, expresses the inner-demarcation line in the city where after and before a subcenter moves outward. The distance from the new subcenter to the new inner-demarcation line is longer than that from the old subcenter to the old inner-demarcation line, since $\frac{dz(\theta)}{dm} = \frac{1}{2-\theta} > 0$, for $\theta \in [0,1]$, where $z(\theta)$ denotes the distance from subcenter to inner-demarcation line. And the distance from new subcenter to the new SBD-urban fringe (GW' line or G'W' line) should be equal to the distance from the old subcenter to the old SBD-urban fringe (GW line or G'W line), since both are always equal to b. Thus, it is concluded that the new SBD-area can not contract at any direction: namely, if we clip the old SBD-area from the paper and paste it in the new one, we find that the old SBD-area is completely included in the new one, although the old and the new SBD-urban fringes coincide.

Now we have demonstrated that both CBD-area and SBD-area expand with subcenter location, but it contradicts the presumption that $\frac{dN}{dm} < 0$. Thus it must be that $\frac{dN}{dm} > 0$. Table 3-1 summarizes the results of comparative statics for case A.

Additionally, in an open city, the location of urban development is planned by city government through maximizing social surplus (TDR (Total Differential Rents)), hence, the effects of subcenter location on TDR, as an important index of evaluating urban policy, should be examined. From the fact that people in CBD area do not move regardless of subcenter location, we know the TDR in CBD area is zero. And from Figure 3-5, we know that with subcenter location moving outward, SBD area expands and every point in previous SBD area can be found in the new one, which means bid rent at every location in previous SBD area can be found in new one. Thus, we can conclude that TDR in SBD are increases with subcenter location.

3-3-2 Analysis for case B

The procedure for the analysis for case B is the same as the above. Differentiating (5) (16) (17) and (18') with respect to each parameter and slightly arranging them, we have:

effects of k:

$$\frac{dH_B}{dk} = \frac{db}{dk} = -\frac{b}{k} < 0. \tag{3-32}$$

$$\frac{dE}{dk} = \frac{b}{(1-\theta)k} > 0, \qquad \theta \in [\theta_1, \theta_2]. \tag{3-33}$$

$$\frac{dN}{dk} = -\frac{N + (2\pi - 4)\int_0^b \frac{t}{q}\,dt + 8\int_0^{\frac{m}{2}} \frac{t}{q}dt + 2\int_0^b \frac{t}{m}\frac{t}{q}\,dt}{k} < 0. \tag{3-34}$$

effects of y:

$$\frac{dH_B}{dy} = \frac{db}{dy} = \frac{1}{k} > 0. \tag{3-35}$$

$$\frac{dE}{dk} = -\frac{1}{(1-\theta)k} < 0, \qquad \theta \in [\theta_1, \theta_2]. \tag{3-36}$$

$$\frac{dN}{dy} = \frac{(2\pi - 4)\int_0^b \frac{1}{q}\,dt + 8\int_0^{\frac{m}{2}} \frac{1}{q}dt + 2\int_0^b \frac{t}{m}\frac{1}{q}\,dt}{k} > 0. \tag{3-37}$$

effects of u:

$$\frac{dH_B}{du} = \frac{db}{du} = -\frac{1}{u_x(b)k} < 0. \tag{3-38}$$

$$\frac{dE}{du} = \frac{1}{(1-\theta)u_x(b)k} > 0, \qquad \theta \in [\theta_1, \theta_2]. \tag{3-39}$$

$$\frac{dN}{du} = -\frac{(2\pi - 4)\int_0^b \frac{1}{u_x(b)q}\,dt + 8\int_0^{\frac{m}{2}} \frac{1}{u_x(b)q}dt + 2\int_0^b \frac{t}{m}\frac{1}{u_x(b)q}\,dt}{k} < 0. \tag{3-40}$$

effects of S:

$$\frac{dH_B}{dS} = \frac{db}{dS} = -\frac{q(b)}{k} < 0. \tag{3-41}$$

$$\frac{dE}{dS} = \frac{q(b)}{(1-\theta)k} > 0, \qquad \theta \in \left[\theta_1, \theta_2\right]. \tag{3-42}$$

$$\frac{dN}{dS} = -\frac{(2\pi - 4)b + 4m + \frac{b^2}{m}}{k} < 0. \tag{3-43}$$

effects of m:

$$\frac{db}{dm} = 0. \tag{3-44}$$

$$\frac{dE}{dm} = \frac{1}{1-\theta} > 0, \qquad \theta \in \left[\theta_1, \theta_2\right]. \tag{3-45}$$

$$\frac{dH_B}{dm} = 1 - \theta > 0, \qquad \theta \in \left[0, \theta_2\right]. \tag{3-46}$$

$$\frac{dN}{dm} \gtrless 0. \tag{3-47}$$

effects on r and R:

$$\frac{\partial r}{\partial k} < 0, \quad \frac{\partial r}{\partial y} > 0, \quad \frac{\partial r}{\partial u} < 0, \quad \frac{\partial r}{\partial S} = 0, \quad \frac{\partial r}{\partial m} = 0,$$

$$\frac{\partial R}{\partial k} < 0, \quad \frac{\partial R}{\partial y} > 0, \quad \frac{\partial R}{\partial u} < 0, \quad \frac{\partial R}{\partial S} = 0, \quad \frac{\partial R}{\partial m} = \begin{cases} < 0, & t < m \\ > 0, & t > m \end{cases}. \tag{3-48}$$

The effect of subcenter location on population is ambiguous in (3-47), but we can determine the sign of $\frac{dN}{dm}$. We first suppose that $\frac{dN}{dm} < 0$. Since all other parameters are unchanged, a decrease in population means that city size must reduce in either CBD-area or SBD-area or both areas. From (3-31) and (3-44) it is demonstrated that CBD-area expands with subcenter location. It is also proved that SBD-area is expanded with subcenter location in the following way.

In Figure 3-6, when a subcenter moves from A to A', the inner-demarcation line moves from DD' curve to EE' curve, inner-SBD-urban fringe from DG (or D'G' curve) to EF (or E'F' curve), and the outer-SBD-urban fringe moves from GW (or G'W curve) to FW' (or F'W' curve). In the same way as in case A, we see that the distance from the new subcenter to both new SBD-urban fringes is equal to that from the old subcenter to both old SBD-urban fringes, which should be equal to b. Next we differentiate $v(\theta) = \frac{m}{2-\theta}$ for $\theta \in [0, \theta_1]$ with respect to m to obtain:

$$\frac{dv(\theta)}{dm} = \frac{1}{2-\theta} > 0, \qquad \theta \in [0, \theta_1]. \tag{3-49}$$

From (3-49), it is seen that the distance from the new subcenter to the new inner-demarcation line is longer than that from the old subcenter to the old inner-demarcation line, and thus new SBD-area expands in the direction toward CBD-area. Since its area is unchanged in other directions, the new SBD-area is to be expanded.

To sum up, it follows that both CBD-area and SBD-area expand with subcenter, but it contradicts the presumption that $\frac{dN}{dm} < 0$. Therefore it must hold that $\frac{dN}{dm} > 0$. The results for case B so far are summarized in Table 3-2.

Using the same way in analyzing case A, we can also demonstrate that TDR increases with subcenter location in case B.

3-4 Concluding Remarks

In this chapter, we built two main models of an open city with a subcenter. The urban shape for each case was discussed in detail and the comparative static analyses were performed. In this final section, we summarize the results. As shown in the first row in both Tables 4.1 and

4.2, It is found that an increase in transport cost causes city population to decrease and urban fringes to move inward. As a result of out-migration, the bid rent is lowered at each location in the city.

The effect of income level is opposite to the effect of commuting cost: a higher income level causes the population to increase, city size to expand, and land rent to be raised at every location.

Both population and city size decrease when the utility level is heightened, and so does the rent at every location.

An increase in the agriculture rent necessarily contracts the urban fringe, and then decreases the population, but it keeps the rent in a city unchanged.

Since a subcenter is established in an existing Alonso-type city, the CBD urban fringe and the rent in the CBD area are not influenced by subcenter establishment in the open-city setting. But the SBD-urban fringe and the rent in SBD area are affected by the location of new worksite. As shown in the last line in both Tables 3-1 and 3-2, a further subcenter location moves the SBD-urban fringe outward, then it necessarily increases the population in the city since the utility level is kept constant. The effect of subcenter location on the rent in the SBD area is divided into two parts according to the subcenter location and residents' location. If residential location is closer than the subcenter to the CBD, the bid rent at that location will be decreased by a further subcenter location. If residential location is further than the subcenter, the rent at that location will be increased when subcenter moves outward.

In comparing the present results with results in Sasaki's [1987a] (see also Wheaton's [1974], David Pines & Efraim Sadka [1986] and Brueckner [1987]), it is found, as in Zhang and Sasaki [1995] for the analysis of a closed city, that the establishment of a subcenter does not alter the effect of parameters in a monocentric urban model.

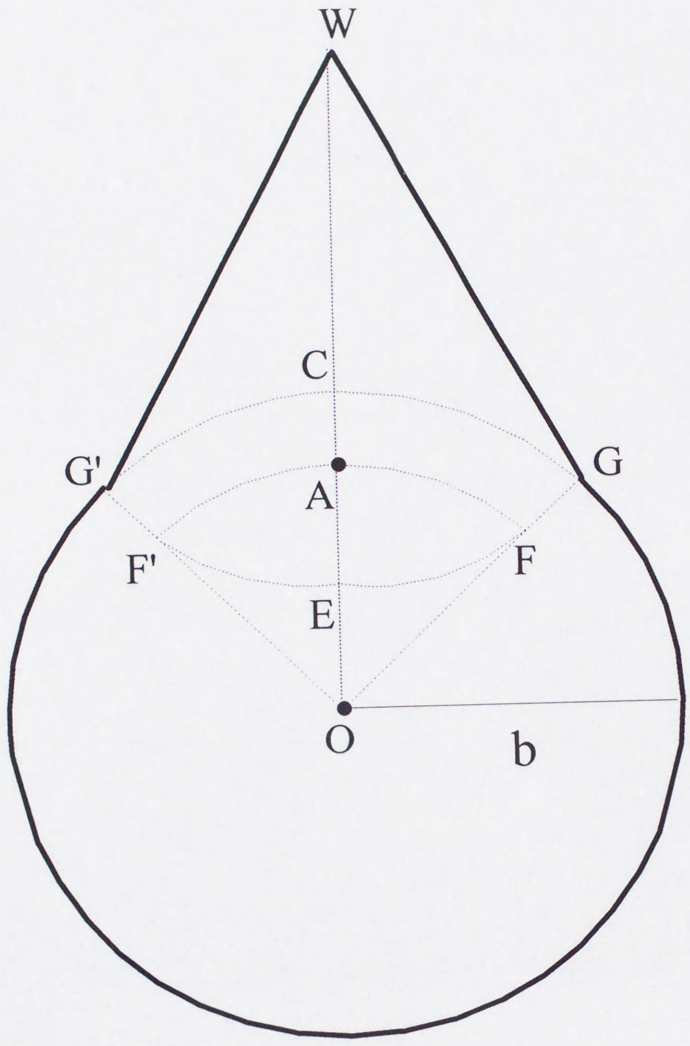


Figure 3-1 Urban shape of case A for a duo-centric open city

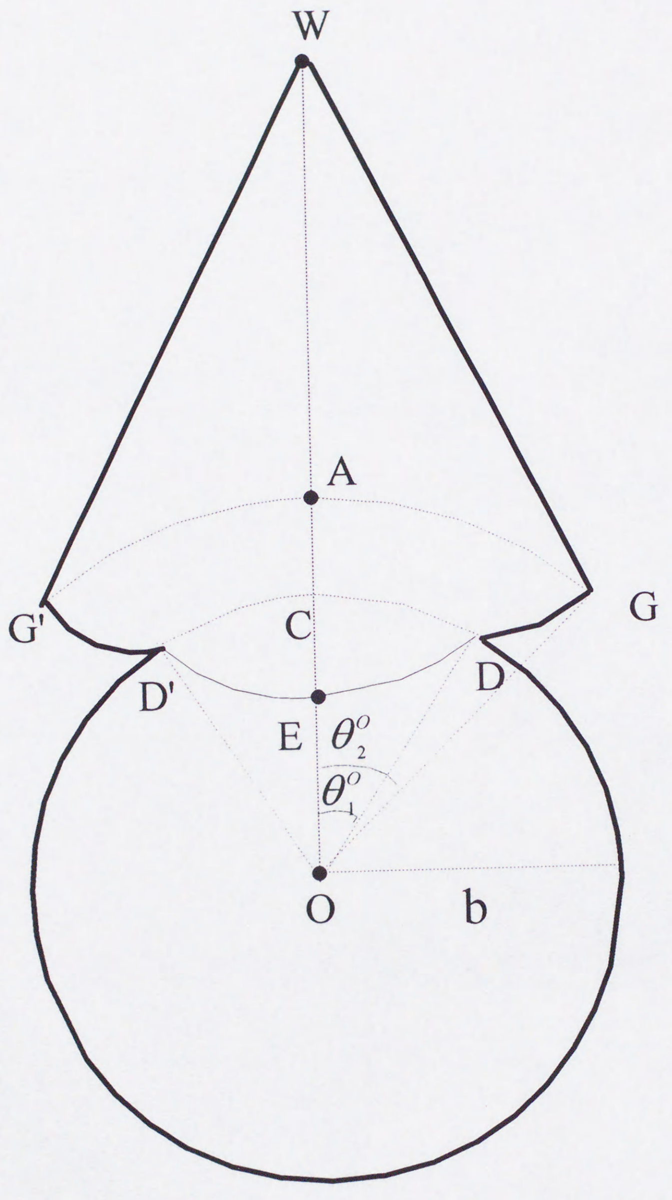


Figure 3-2 Urban shape of case B for a duo-centric open city

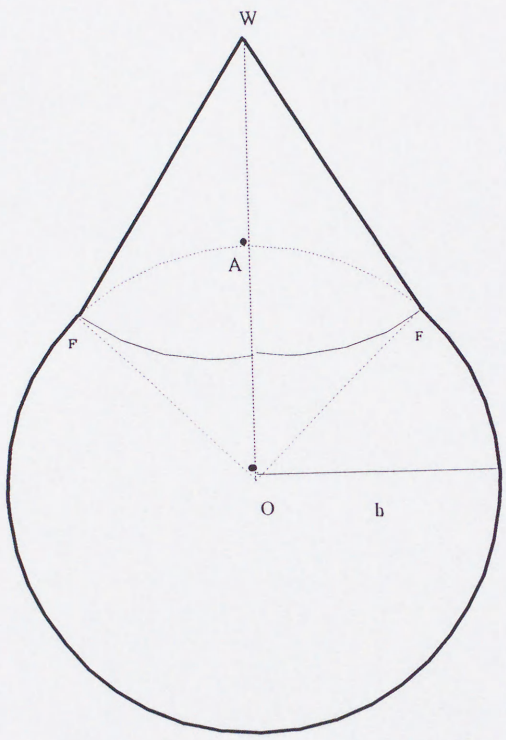


Figure 3-3 Urban shape of open city where CBD-urban fringe is equal to subcenter location

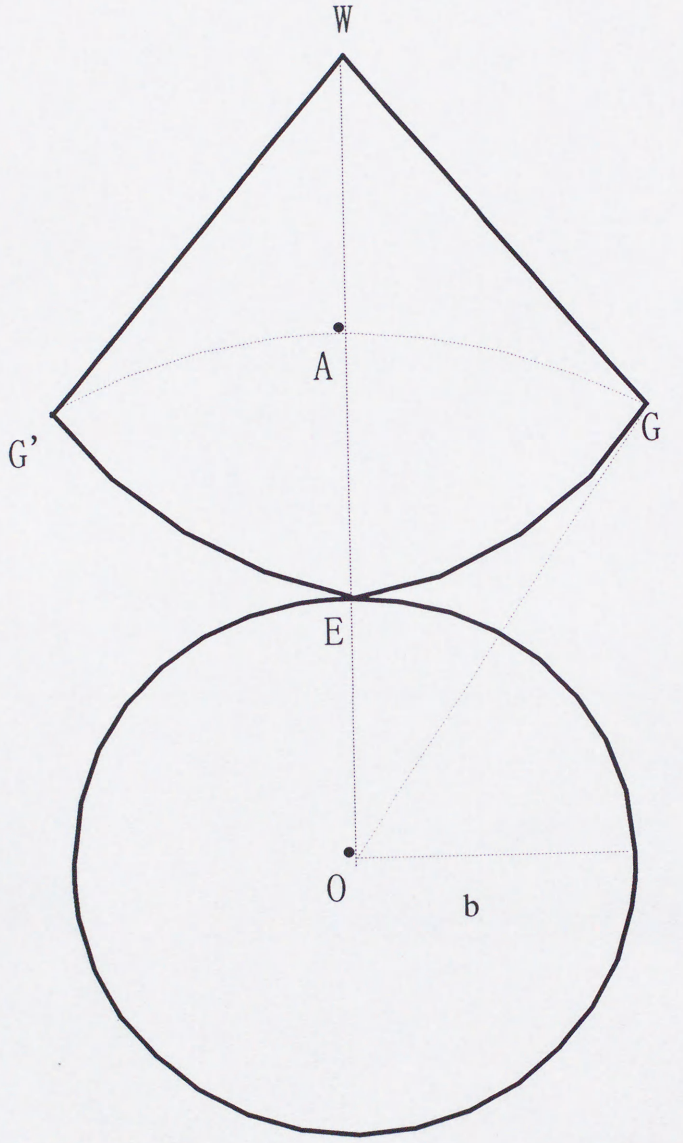


Figure 3-4 Urban shape of an open city where subcenter is created on its maximum subcenter location

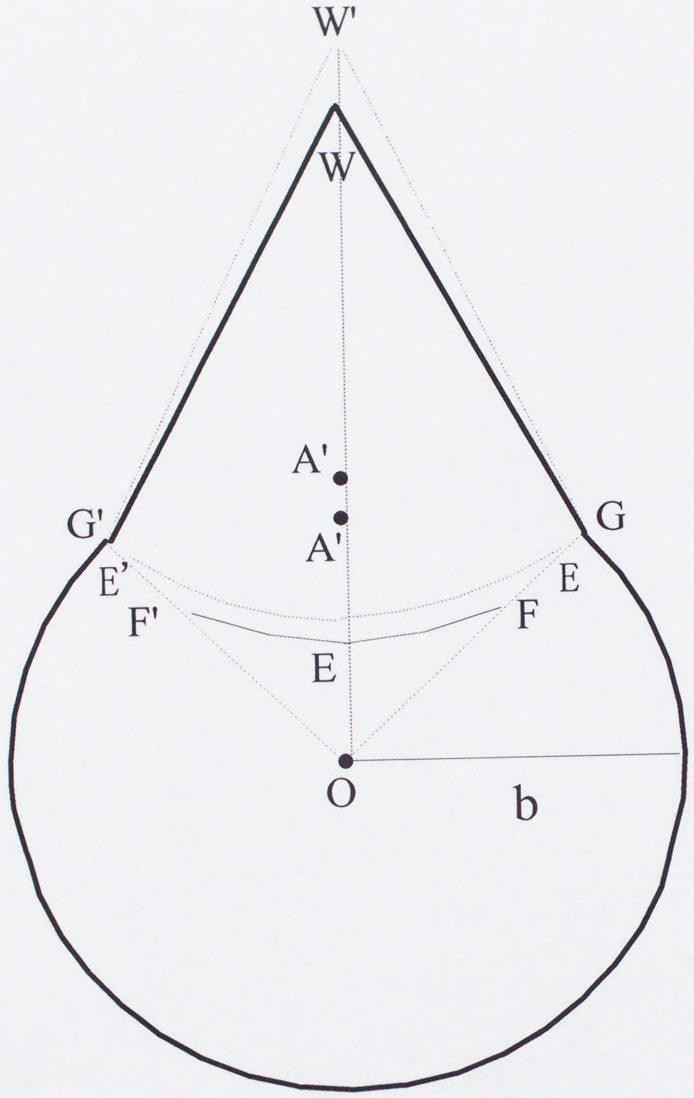


Figure 3-5

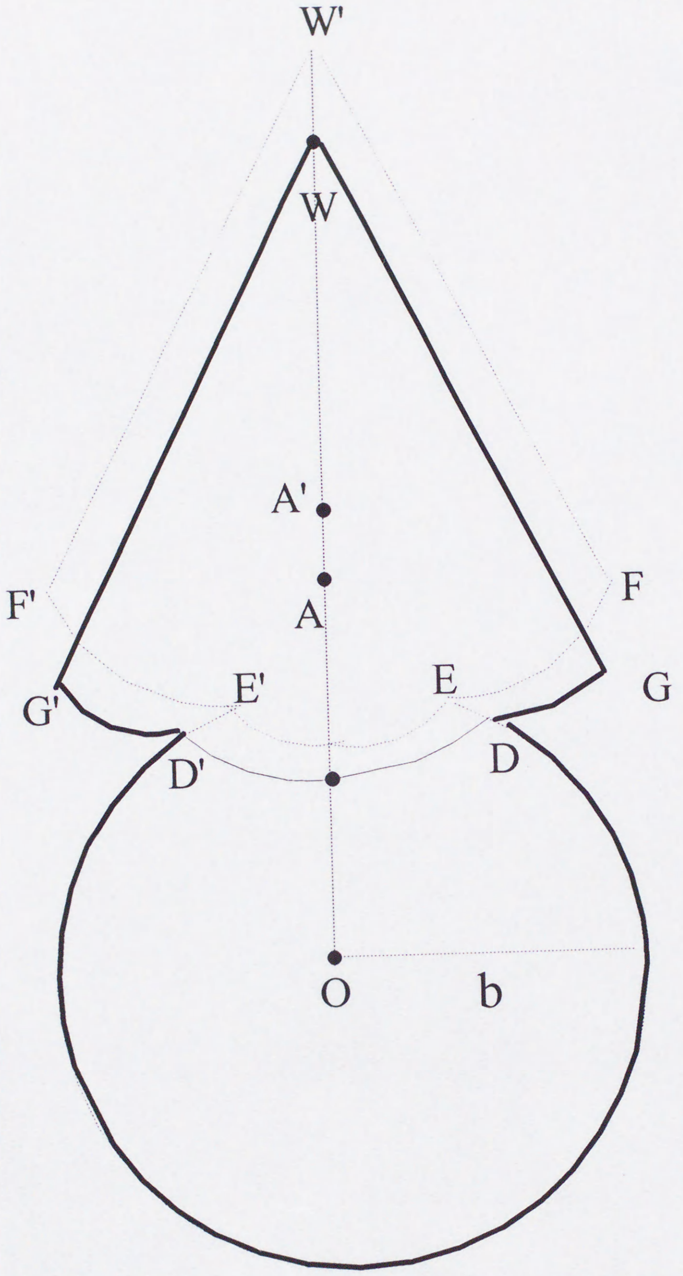


Figure 3-6

Table 3-1. Results of comparative static analysis of case A for an open city with a subcenter

Effect on:	N	b	H	r	R
Increase in					
k	−	−	−	−	−
y	+	+	+	+	+
u	−	−	−	−	−
S	−	−	−	0	0
m	+	0	+	0	− $(t < m)$ + $(t > m)$

Table 3-2. Results of comparative static analysis of case B for an open city with a subcenter

Effect on:	N	b	E	H	r	R
Increase in						
k	−	−	+	−	−	−
y	+	+	−	+	+	+
u	−	−	+	−	−	−
S	−	−	+	−	0	0
m	+	0	+	+	0	− $(t < m)$ + $(t > m)$

Chapter 4

Model of edge city

4-1 Introduction

Edge cities, as a new phenomenon of suburbanization, are booming in modern cities in North America, Europe and Asian. Unlike traditional suburbanization or urban sprawl, edge city is developed by large scaled private developers or real estate speculators. As a counterpart of core city, edge city competes with it in land and labor market so as to alter urban land use pattern and cause a decay in core city. With formation of edge city, a large number of skilled labor forces or specialists are attracted to concentrate there, so edge city also presents a perspective on how segregation by income or social-economic class occurs.

Most of literature (see Ogawa & Fujita [1980], Fujita & Ogawa [1982], Helsley & Sullivan [1984], White [1976,1988], Wieand [1987], Sasaki [1990], Yinger [1992], Ross & Yinger [1995] and Zhang & Sasaki [1997]) have discussed the increasing role of the second employment worksite, in which both CBD and SBD firms are small so that they have no market power in both labor and land markets. Henderson & Slade [1993], Henderson & Mitra [1996] develop a general approach for studying edge city, in which the second employment center is designed by a developer who holds many firms and modern urban landscape is determined by strategic choices adopted by the developer who design reagglomerations of people.

In another work performed by Fujita, Thisse & Zenou [1997], they assume that the second employment worksite is planned by a large agent who engages in common production activities or real estate. Their model is more general approach in that wage rate is endogenously determined, although labor demand function is specified. However, in the models of both Henderson et al and Fujita et al, residential land consumption is assumed to be independent of their locations, so the results can not parallel to the those in most of literature on the residential location analysis.

In this chapter, we model the edge city phenomenon. We examine the strategic choices by one edge city for location vis-à-vis the core city. Drawing above authors' lessons, we treat the behavior of residents as well as stated in basic urban model. We consider both labor and land markets in the model by referring Sasaki & Kaiyama [1990]'s work. In the last section, we take a comparison with existing works.

4-2 Model

4-2-1 Model of port city

There exist a historical port city in a given linear open region (Fig 4-1). We call it as port city since its left side of original point faces sea or mountain. On the other word, geographically, the left side can not be developed any more. We assume that all job opportunities are concentrated in CBD. In the whole economic system, there are totally three agents: absentee landlords, consumers and firms. First, we introduce the behavior of each agent.

1. Behavior of absentee landlords

Absentee landlords as owners of land, lend their land to the highest bidder. The total land rents are their profits.

2. Behavior of firms

We assume that there are infinite number of firms who locate in CBD area. They hold an identical production technology and employ land LP and labor NP to produce the same export goods. We give firms' production function in the following form:

$$Q_P = Q_P(LP, NP) \tag{4-1}$$

where the subscription P (hereafter) denotes port city.

We assume the production market is perfect competition and the firms' production function is linear homogeneous. Thus, the production for one unit land can be written as:

$$q_P^f = q_P^f(1, \Delta_P) \tag{4-2}$$

where Δ_P denotes $\frac{NP}{LP}$, which means that for unit land, how many labors are input. Superscription f (hereafter) denotes firms.

For making workers get the same income in CBD area, wage rate function W_P of a firm locating at t is assumed as:

$$W_P(t) = WP - c_P^c t \tag{4-3}$$

where WP denotes the basic wage rate at point 0, c_P^c expresses commuting cost of a round trip per capita. The superscription c (hereafter) expresses consumer.

Products are carried to point 0 and export to national market. Thus, for a firm locating at t, its profit function Π_P of one unit land can be expressed as:

$$\Pi_P = (P_P - c_P^f t)q_P^f - (W_P - c_P^c t)\Delta_P - r_P^f \tag{4-4}$$

where P_P denotes the fixed market price of export goods, c_P^f one unit distant transportation cost of one unit product, r_P^f is the market land price at t. Since each firm acquires the same profit (i.e., zero) in the equilibrium, the market rent in the industrial district coincides with the bid rent of a firm associated with the zero profit level attained in the equilibrium, i.e.,

$$r_P^f(t) = \max_{\Delta_P} (P_P - c_P^f t)q_P(\Delta_P(t)) - (WP - c_P^c t)\Delta_P(t) \tag{4-5}$$

Solving (4-5), we obtain optimal values of Δ_P and r_P^f:

$$\begin{aligned} \Delta_P &= \Delta_P(P_P, WP, c_P^f, c_P^c, t) \\ r_P^f &= r_P^f(P_P, WP, c_P^c, c_P^f, t) \end{aligned} \tag{4-6}$$

3. Behavior of consumers

There are NP residents who hold identical preference living in the area surround industrial district and commuting to CBD area. Residential utility function relies on two normal goods: composite goods z_P and land consumption q_P^c. For a resident who lives at t and works at t_f, his utility-maximization problem is defined as following:

$$\max_{z_P, q_P^c} u = u\left(z_P, q_P^c\right)$$

$$s.t. \quad (WP - c_P^c t_f) = z_P + r_P^c(t) q_P^c + c_P^c (t - t_f). \tag{4-7}$$

arranging (4-7), we have:

$$\max_{z_P, q_P^c} u = u\left(z_P, q_P^c\right)$$

$$s.t. \quad WP = z_P + r_P^c(t) q_P^c + c_P^c t \tag{4-7'}$$

Moreover, we obtain residential bid rent function:

$$r_P^c(t) = \max_{z_P, q_P^c} \frac{WP - z_P - c_P^c t}{q_P^c}$$

$$s.t. u = u\left(z_P, q_P^c\right). \tag{4-8}$$

solving (4-7) we obtain:

$$\begin{aligned} z_P &= z_P(u, t, c_P^c, WP) \\ q_P^c &= q_P^c(u, t, c_P^c, WP) \\ r_P^c &= r_P^c(u, t, c_P^c, WP). \end{aligned} \tag{4-9}$$

4. Equilibrium

Within above framework, we totally have four variables: boundary of industrial area a, urban fringe A, population NP and wage rate WP, which are simultaneously determined in the following four equilibrium conditions:

$$r_P^c(u, A, c_P^c, WP) = S \tag{4-10}$$

$$r_P^c(u, a, c_P^c, WP) = r_P^f(P_P, WP, c_P^c, c_P^f, a) \tag{4-11}$$

$$\int_0^a \Delta_P(P_P, WP, c_P^f, t)dt = NP \tag{4-12}$$

$$\int_a^A \frac{1}{q(u, t, c_P^c, WP)} dt = NP \tag{4-13}$$

where S denotes agricultural land rent. (4-10) and (4-11) respectively stand for that the bid rents at the boundary between any two adjacent land users should be same. (4-12) and (4-13) respectively denote the condition of full employment and population condition.

4-2-2 Model of edge city

In edge city's economic system, we have four agents: absentee landlords, producers, residents and developer. The regional spatial structure is shown in Figure 4-2. T_E denotes the location of edge city, b the boundary between industrial and living areas in port city, B the boundary between two cities, D and F respectively, the left and right boundary of industrial district in edge city and G expresses the edge city's urban fringe on the right side. Next, we will introduce each agent's behavior and see how the new equilibrium achieves. The behavior of absentee landlords is assumed to identical in the region, which has been stated in above section, so the explanation is omitted here.

1. Behavior of producer in edge city

Producers in edge city also use land LE and labor NE to produce a kind of export goods, but production activity is different from those in port city. The differences are expressed as following several aspects: (1) they produce a kind of goods different from port city. (2) The production activity in edge city is influenced by port city's total production quantity, i.e., the larger the total production quantity is in port city, the higher the production activity is in edge city, which corresponds the fact that high agglomerative economy in core city, in great extend, affects economic situation in periphery of the city. The externality of the agglomerative economy is measured by total port city's population in the model. (3) Production activity in edge city is also affected by situation of public facility, which means the more convenient public facility produces higher production activities. So another externality, quantity of capital invested on public facility by developer is also introduced. (4) Since firms efficiency depends on the given volume of undecayed messages received from other firms, and the messages are costly to purchase and move through space. Corresponding the fact, we assume that edge city's production activity is varied with the quality of information coming from core city, and the information is decayed with distance [see Henderson 1996].

According to the behavior of producers in edge city stated above, we can give their production function Q_E as:

$$Q_E = NP^{\alpha 1} K^{\alpha 2} T_E^{-\alpha 3} Q_E^f(NE, LE) \qquad \alpha 1, \alpha 2, \alpha 3 > 0 \tag{4-14}$$

where K denotes capital investment on public facility, $\alpha 1$, $\alpha 2$ and $\alpha 3$ respectively, stand for the degree of economies of scale associated with total port city's production quantity, capital investment and information received. Subscription E (hereafter) denotes edge city. From assumptions in behavior of producers, we know that $\frac{\partial Q_E}{\partial NP} > 0$, $\frac{\partial Q_E}{\partial K} > 0$, $\frac{\partial Q_E}{\partial T_E} < 0$, and if $NP = 0$, then $Q_E = 0$. (4-13) is also assumed to be homogeneous linear. So the production for one unit land can be written as:

$$q_E^f = NP^{\alpha 1} K^{\alpha 2} T_E^{-\alpha 3} q_E^f(\Delta_E, 1) \tag{4-15}$$

where Δ_E denotes $\frac{NE}{LE}$, which means that for one unit land, how many labors are input.

As well as treated in port city, for making workers get the same income in edge city's industrial area, wage rate function W_E of a firm locating at t is assumed as:

$$W_E(t) = WE - c_E^c|t - T_E| \tag{4-16}$$

where WE denotes the basic wage rate at point T_E, c_E^c expresses commuting cost of a round trip per capita in edge city.

The national market of edge city's product is also assumed to be perfect competitive. The goods produced in edge city are carried to the center of edge city and exported from there. Thus, for firm which locates at t, its profit function for one unit land can be written as:

$$\Pi_E = \left(P_E - c_E^f|t - T_E|\right)q_E^f - \left(W_E - c_E^c|t - T_E|\right)\Delta_E - r_E^f \tag{4-17}$$

where P_E denotes the fixed market price of export goods, c_E^f one unit distant transportation cost of one unit product, r_E^f is the market land price at t. Each firm should acquire the same profit (i.e., zero) in the equilibrium since national market of product is perfect competitive. The market land price in the industrial district must coincide with the bid rent of a firm associated with the zero profit level attained in the equilibrium. Thus, let the left side of (4-17) be zero and arrange it, we obtain the firms' bid rent function:

$$r_E^f(t) = \max_{\Delta_E} \left(P_E - c_E^f|t - T_E|\right)q_E^f - \left(W_E - c_E^c|t - T_E|\right)\Delta_E \tag{4-18}$$

Solving the problem in (4-18), the optimal values of Δ_E and r_E^f can be obtained:

$$\begin{aligned} \Delta_E &= \Delta_E(P_E, WE, T_E, t, c_E^f, c_E^c) \\ r_E^f &= r_E^f(P_E, WE, T_E, t, c_E^f, c_E^c) \end{aligned} \tag{4-19}$$

2. Behavior of consumers in edge city

Behavior of consumers in the given region is assumed to be identical. The unique difference between consumers of two groups is their commuting pattern. Unlike port city's consumers' purely commuting towards left side, after edge city is established, some consumers who locate near the left side of T_E in previous port city will alter their work place to the new employment center. Thus, in edge city, there exist a right up-towards bid rent curve and peaks at location T_E. Keeping all assumptions concerning to the behavior of consumers unchanging, we can write the bid rent function of a resident locating at t in edge city as:

$$r_E^c(t) = \max_{z_E, q_E^c} \frac{WE - z_E - c_E^c |t - T_E|}{q_E^c}$$

$$s.t. \quad u = u(z_E, q_E^c). \tag{4-20}$$

Solving (4-7), we obtain:

$$\begin{aligned} z_E &= z_E(u, T_E, t, c_E^c, WE) \\ q_E^c &= q_E^c(u, T_E, t, c_E^c, WE) \\ r_E^c &= r_E^c(u, T_E, t, c_E^c, WE). \end{aligned} \tag{4-21}$$

3. Behavior of developer

The developer treated here is an agent of purely engaging in estate, he borrows lands from absentee landlords and establishes or renovates public facilities on them, then lends the lands to the agents who want to locate there. So the benefit for developer is pure land rents, the cost for him includes two parts: land rents pay to absentee landlords and capital investment on public facilities. So developer's action is how to strategically choose edge city's location and quantity of capital invested on public facilities to maximize his profits. For instance, he may create edge city at somewhere in previous port city, even in CBD of port city, the production activity in edge city will be higher because of avoiding information decay, so that he can pose higher land rents from lessees who want to locate in his developed area. But on the other hand, in order to obtain the land from absentee landlords, he has to pay higher land costs to bid current users out. Oppositely, if he develops an edge city in the rural land far away from port city for avoiding high land costs, he has to burden the risk that over-decayed messages probably forces firms to give up movement or relocation. If that case happened, no movement of firms causes non-employment opportunities in new developed area. Thereby, he obtains land rent income neither from firms, nor from residents.

4.Equilibrium

Henderson [1996] considered six equilibrium cases that correspond six land use patterns, in which three cases result in happening of the cases of monocentric city. In order to highlight edge city's independency, the case of monocentric city is ignored in the context. Now, we give developer's profit function Π_D:

$$\Pi_D = \max_{T_E, K} \int_B^D r_E^c dt + \int_F^G r_E^c dt + \int_D^{T_E} r_E^f dt + \int_{T_E}^F r_E^f dt - R - kK \tag{4-22}$$

where k is the price of capital and R denotes the cost on purchasing land. The first four terms in the right side of (4-22) denotes the land rent revenue from both firms and residents, and the last term stands for the cost on capital investment.

Solving the problem shown in (4-22), we obtain two first-order conditions:

$$\frac{\partial \Pi_D}{\partial T_E} = 0 \tag{4-23}$$

$$\frac{\partial \Pi_D}{\partial K} = 0 \tag{4-24}$$

It is noted here that according to location of points B (the left urban fringe of edge city) and G (the right urban fringe of edge city), the value of R is different. On the other word, B and G are probably located both in industrial, living or rural areas, or separately located in different area so that the cost of purchasing land is different. Totally, combination of B and G yields six regional spatial structures. In order to correctly write developer's profit function, we have to discuss them respectively.

Next, we see sub-case 1, which means $G > A$ in equilibrium. According to the value of B, we may classify the case into three lower ranking cases. We see the case of $G > A$ and $B = A$ (sub-case 1A) first. The description for Sub-case 1A is shown in Figure 4-3. If the degree ($\alpha 1$)

of economies of scale associated with total port city's output is relative large or the degree ($\alpha 3$) of economies of scale associated with information decay is relative small in edge city's production activity, developer prefer to create edge city at somewhere far from the core city to pursue higher profit. If it is true, this case might be happened. In this case, the value of R should be:

$$R = (G - A)S \tag{4-25}$$

Sub-case 1B (Figure 4-4):

Sub-case 1B expresses that $G > A$ and $a < B < A$. R's value for this case will be:

$$R = (G - A)S + \int_B^A r_P^c dt \tag{4-26}$$

Sub-case 1C (Figure 4-5):

Sub-case 1C presents that $G > A$ and $B < a$, which means that after edge city is established, the size of port city contracts in a great extent. If the case is true, R's value is:

$$R = (G - A)S + \int_a^A r_P^c dt + \int_B^a r_P^f dt \tag{4-27}$$

Next, we see the case of $a < G < A$ (sub-case 2). In this case, since value of B could not exceed that of G, we have two lower ranking cases.

Sub-case 2A:

Sub-case 2A denotes that $a < G < A$ and $a < B < A$. Regional spatial configuration is described in Figure 4-6 (). Developer's costs on purchasing lands for the case should be:

$$R = \int_B^G r_P^c dt \tag{4-28}$$

Sub-case 2B (Figure 4-7):

Sub-case 2B expresses that $a < G < A$ and $B < a$. Figure 4-7 presents the spatial structure for this case. Developer's lands costs R is:

$$R = \int_a^G r_P^c dt + \int_B^a r_P^f dt \tag{4-29}$$

Next, we discuss the last Sub-case (sub-case 3) that $G < a$. This case may be an extreme case, which means that developer creates edge city at somewhere in port city's industrial district and core city seriously decays (refers Figure 4-8). If this case holds in final equilibrium, land costs of developer will be:

$$R = \int_B^G r_P^f dt \tag{4-30}$$

4.1 Equilibrium conditions

Once developer chooses edge city's location and capital investment, equilibrium for sub-case 1A will be determined in the following system:

$$r_P^c(u, B, c_P^c, WP) = S \tag{4-31}$$

$$r_E^c(u, T_E, G, c_E^c, WE) = S \tag{4-32}$$

$$r_P^c(u, b, c_P^c, WP) = r_P^f(P_P, WP, c_P^c, c_P^f, b) \tag{4-33}$$

$$r_E^c(u, T_E, D, c_E^c, WE) = r_E^f(P_E, WE, T_E, D, c_E^f, c_E^c) \tag{4-34}$$

$$r_E^c(u, T_E, F, c_E^c, WE) = r_E^f(P_E, WE, T_E, F, c_E^f, c_E^c) \tag{4-35}$$

$$\int_0^b \Delta_P(P_P, WP, c_P^f, t)dt = NP \tag{4-36}$$

$$\int_D^F \Delta_E(P_E, WE, T_E, t, c_E^f, c_E^c)dt = NE \tag{4-37}$$

$$\int_b^B \frac{1}{q(u,t,c_P^c,WP)}dt = NP \tag{4-38}$$

$$\int_B^D \frac{t}{q_E^c(u,T_E,t,c_E^c,WE)}dt + \int_F^G \frac{t}{q_E^c(u,T_E,t,c_E^c,WE)}dt = NE \tag{4-39}$$

(4-31)-(4-35) respectively stand for that the bid rents at the boundary between any two adjacent land users should be same. (4-36) and (4-37) express full employment condition in core and edge cities respectively. (4-38) and (4-39) respectively describe population condition in core city and edge city. In equilibrium, we totally have nine variables: b, B, D, F, G, NP, NE, WP, and WE, which are simultaneously determined in (4-31) through (4-39). Moreover, by substituting (4-25) into (4-22), we can simultaneously obtain equilibrium values of eleven variables in the whole system from (4-23), (4-24) and (4-31) through (4-39).

As well as done in sub-case 1A, Same nine variables for other cases are determined in (4-32) through (4-34).

$$r_P^c(u,B,c_P^c,WP) = r_E^c(u,T_E,B,c_E^c,WE) \tag{4-40}$$

By respectively substituting (4-25) through (4-30) into (4-22), we will obtain equilibrium values of all eleven variables from the system of consisting of (4-23), (4-24) and (4-32) through (4-40). Based on author's experience, except for numerical simulation, it is difficult to obtain analytic results.

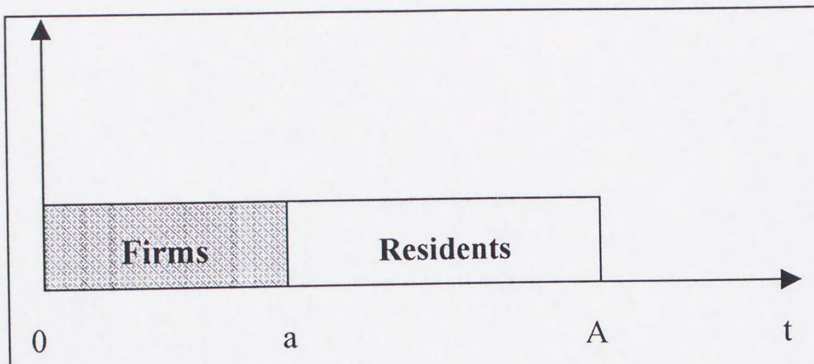
4-3 Comparison to Henderson et al and Fujita et al models

In above section, we have created a model to analyzed edge city phenomenon. Henderson & Mitra [1996] treat developer as a large agent who holds an infinite number of firms. However, several shortcomings are in their work. First, developer's land cost is ignored. For analyzing simply, they assume edge city as a point. However, this is less general than the model presented in this chapter because their model does not reflect the real urban land use and the behavior of speculator stated in Garreau [1991]. Second, residential behavior is not explicitly expounded. Edge city competes labor forces with core city in labor market so endogenous wage rate is required to be introduced into model. Otherwise, there is no incentive for letting residents to move. Third, the assumption of homogeneous labor force can not reflect the fact that many skilled labor or specialists relocate in edge.

Fujita et al [1997]'s model is more general than Henderson & Mitra in that wage rate is endogenously determined in the system. However, as same as Henderson et al work, the assumptions of point edge city location and homogeneous workers make model lack reality.

4-4 Concluding Remarks

In this chapter, along the work performed by Sasaki & Kaiyame [1990], we built an edge city model, in which both labor and land markets are considered. Unlike the behavior of developers expounded in Henderson & Mitra [1995] and Fujita, Thisse & Zenou [1997]'s papers, we treat developer as an agent of purely engaging in real estate. For highlighting edge city's independency, the case of developer's choosing edge city's location in the center of core city is ignored. Meanwhile, following the fact that labor forces hold the different skill in core city and edge city (see , we let wage rates in both cities be determined endogenously. Because of complexity of system, analytic results can only be numerically obtained.



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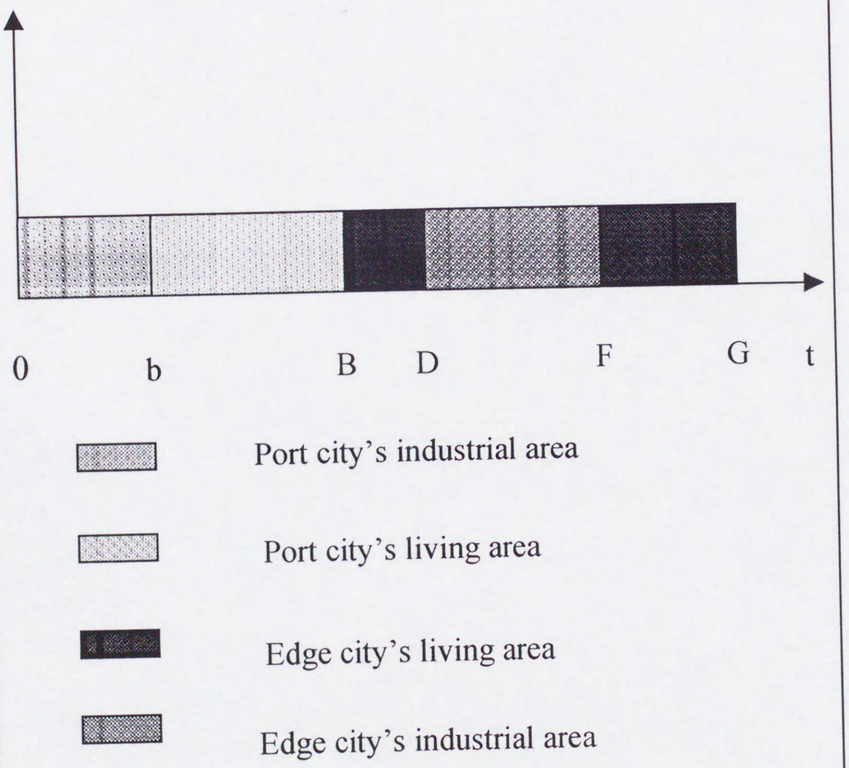

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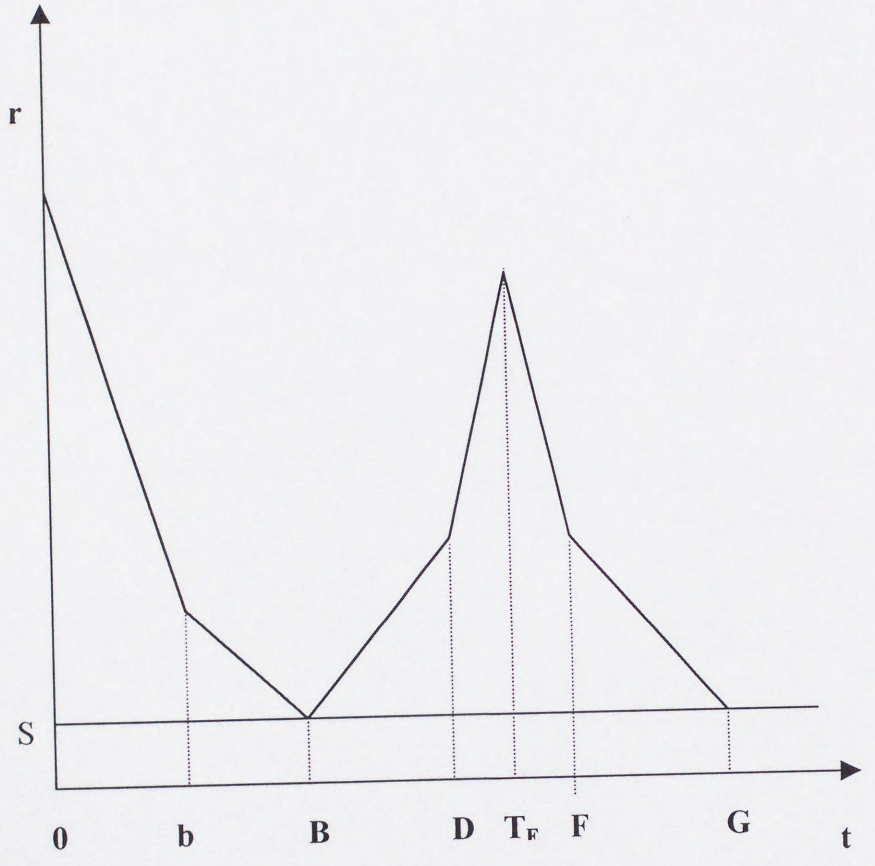

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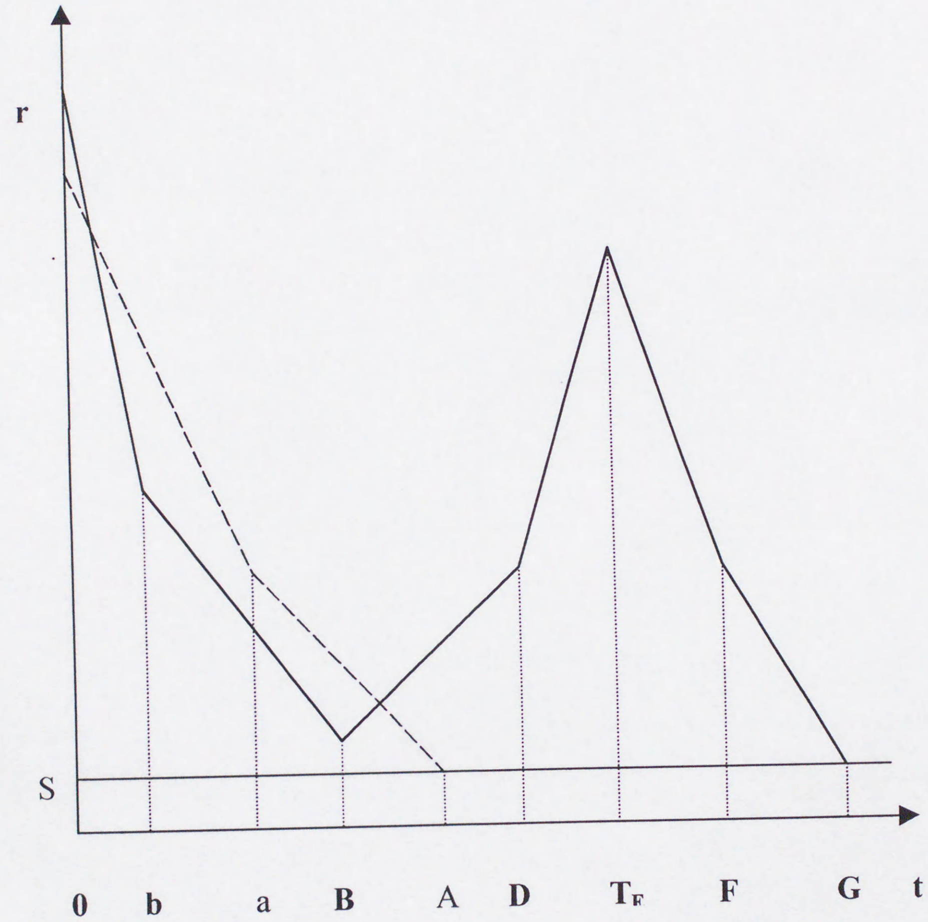

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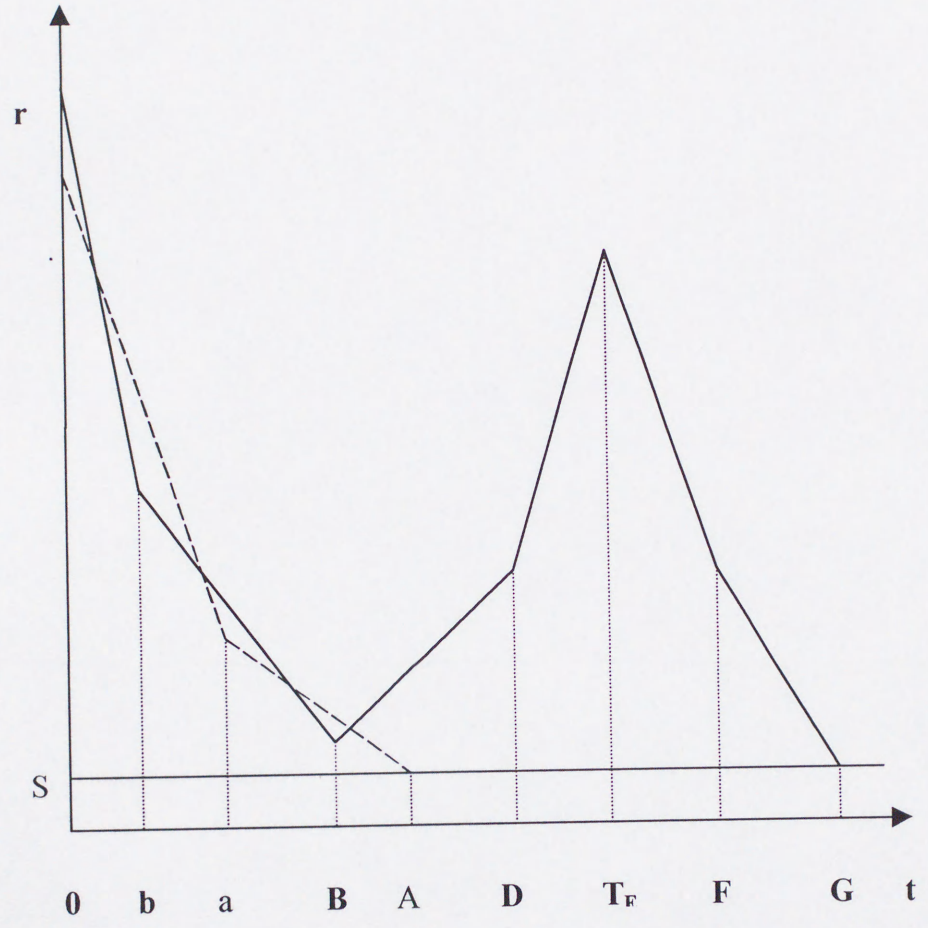

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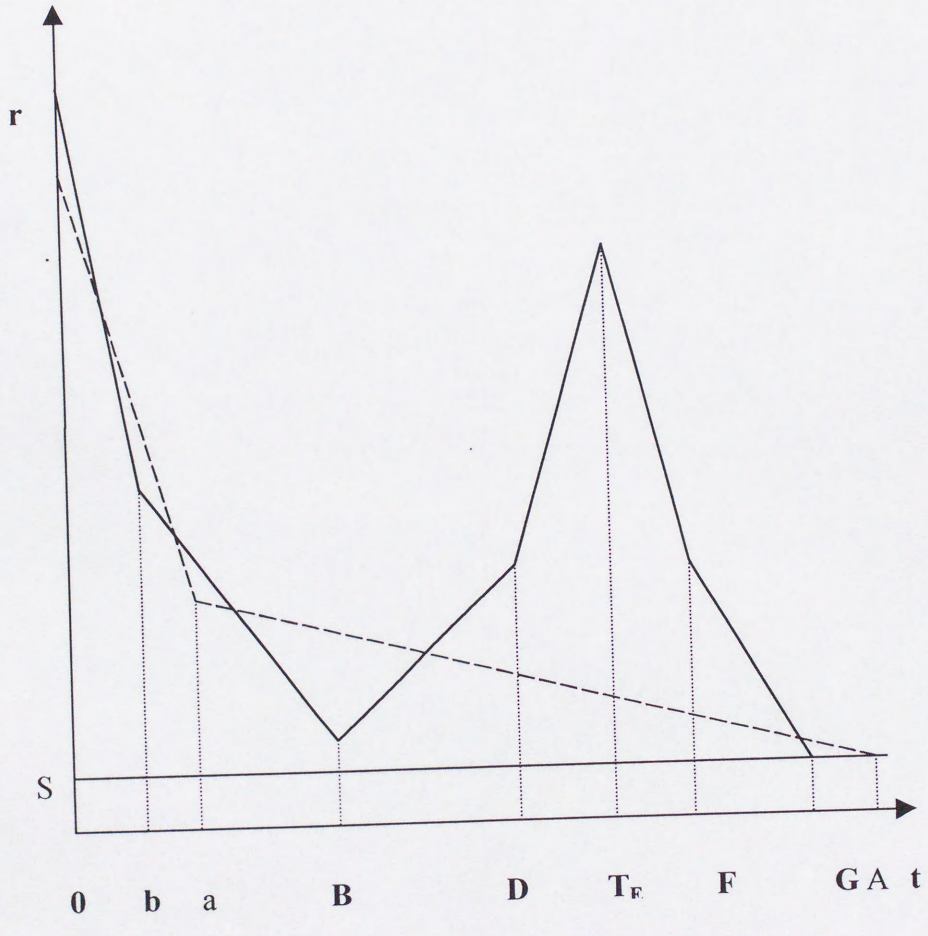

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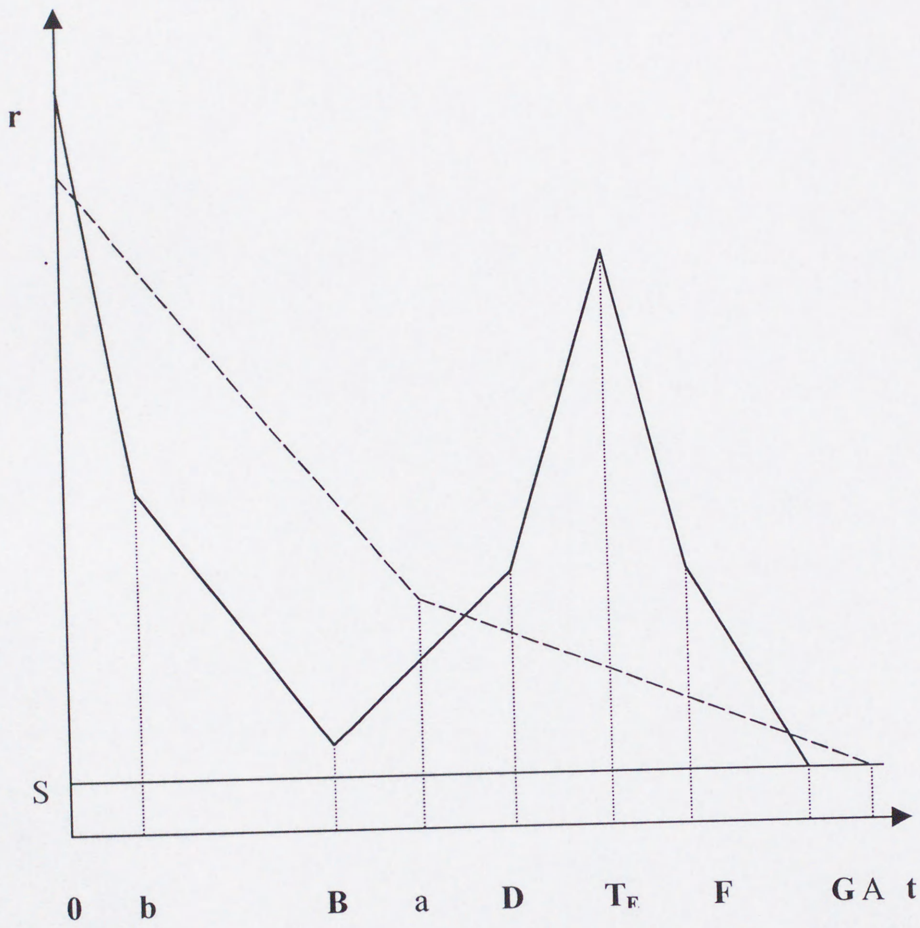

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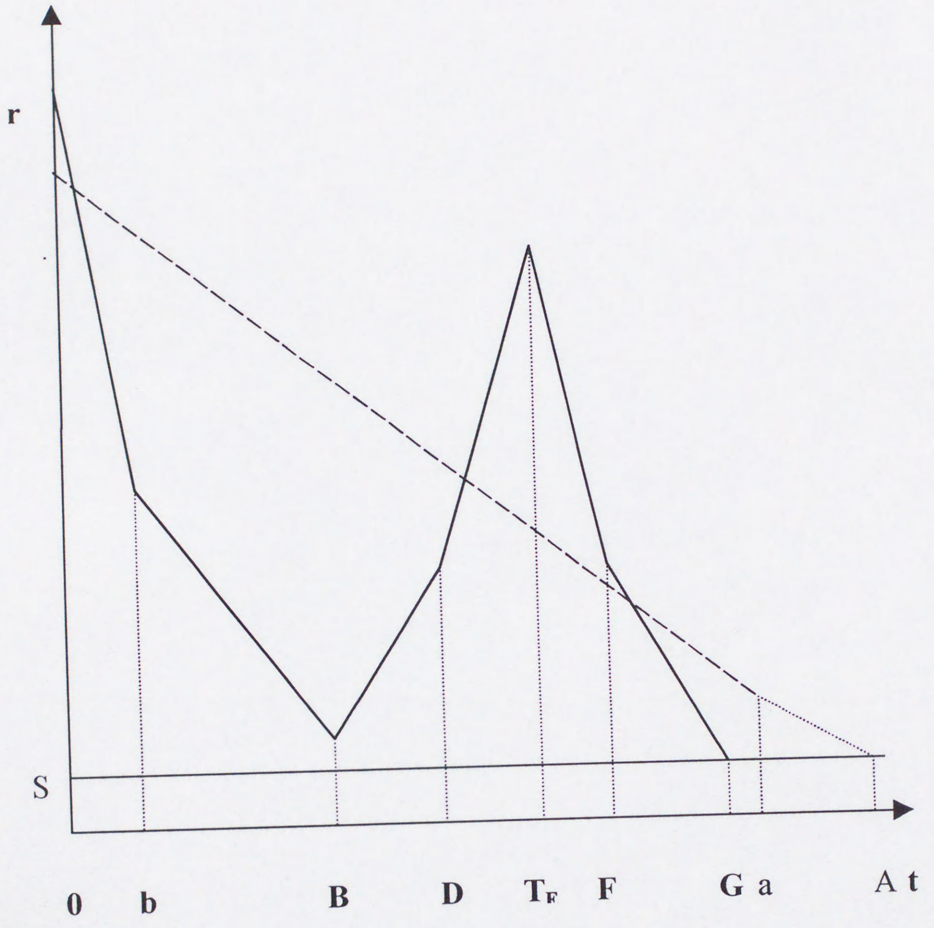

Figure 4-8

Chapter 5

Retrospect and Prospect

5-1 Retrospect

In this paper, we performed a study on suburbanization. Based on mature researches of monocentric city (Alonso [1964]), we built two models to examine how emergency of a subcenter influences urban spatial structure. The model developed Anas and Moses [1979] and Yinger [1992]'s works and performed a comparative static analyses that they did not carry out. Analysis result shows that establishment of subcenter increases residential welfare in a closed city and expands city size in an open city.

Yet, the model remains limited in several respects. First, several restrictive assumptions should be relaxed. For instance, for emphasizing how firms in SBD compete labor forces with those in CBD, the income levels (wage rate) in both CBD-area and SBD-area should be endogenously determined as well as done in White [1976, 1988], Sullivan [1986], Sasaki [1990], Yinger [1992] and Ross & Yinger [1995]. Second, more decision variables should be given to firms, for example, land and labor consumption (see Sullivan [1986], Sasaki [1990] and Ross & Yinger [1995]). The models ignored the behavior of producer, so in some sense, they can not reflect land use pattern existing in the real world. Third, the behavior of agent (government) of establishing subcenter is not expounded explicitly. As an investor, both government's benefits and costs should be analyzed in the model (see Sasaki [1987] and Fujita [1989]).

As another shortcoming of the closed model, because of the complexity of transportation system assumed in the city, the boundary conditions among four cases could not be obtained (see Ogawa & Fujita [1980] and Fujita & Ogawa [1982]).

In this paper, we also established a model to study a new phenomenon of suburbanization, which is booming in modern cities: edge cities. In that model, we drew the lessons of previous subcenter models to consider a full labor market, and land market is competitive among all users. Following facts stated in Garreau [1991], we let income level be determined endogenously to emphasize the phenomenon that after edge city is established, most of skilled workers are attracted to locate there.

However, there still are several shortcomings in the model. First, the model is less general since it is built on a linear region. Second, developer's production activity is not clearly treated. Developer is assumed as an agent of engaging in real estate, so it should carry out production activity such as creation of houses or office spaces. Thus, both benefits and costs on such production activity should be analyzed.

5-2 Prospect

Up to now, most of models are created based on a static economic situation and their analyses are also performed in a static system. Some literature (Miyao [1975], Anas [1978], Mill [1980], Brueckner [1980a, 1980b, 1981a, 1981b, 1982], Brueckner & Burkhard [1981], Fujita [1982], Wheaton [1982a, 1982b] and Bar-Ilan & Strange [1996]) suggests that land development does not only express on the space, but on a time series. In order to examine how cities developed with time series, dynamic analytic approach is necessarily introduced into urban land use model. A general dynamic model, in which maximizing present value of land, as an important index is usually evaluated, is conceptualized in three steps. 1. The time horizon is divided into a set of discrete periods. 2. The decision as when to develop land is based on a comparison between the present value of bid rent for all periods. 3. The amount of land developed within each period must equal to net population growth in that period.

Sasaki & Mun [1996] first created a dynamic model to analyze the formation process of subcenter, in which the whole process is divided into three phases, and developing location and time are determined by maximizing TDR. Their simulation results show that both higher commuting cost and lower commuting cost promote the formation of subcenter.

Dynamic model is more general than static one because both location and timing of development are simultaneously determined. Thus, as a kind of development of the models in the paper, dynamic approach should also be introduced.

Reference

Allinson, G. D., 1979, Suburban Tokyo: A Comparative Study in Politics and Social Change, University of California Press.

Alonso, W., 1964, Location and Land Use: Toward a General Theory of Land Rent, Harvard University, Press, Cambridge, MA.

Anas, A., 1978, Dynamics of urban residential growth, Journal of Urban Economics 5.

Anas, A. & Moses, L. N., 1979, Mode choice, transport structure and urban land use, Journal of Urban Economics , 6, 228-246.

Armstrong, R., 1986 National Trends in Office Construction, Employment and Headquarters Location in U.S. Metropolitan areas, In Daniel, spatial patterns, pp.61-93.

Bar-Ilan, A. & Strange, W. C., 1996, Urban development with lags, Journal of Urban Economics 39, 87-113.

Beckman, M. J., 1958, City Hierarchies and the distribution of city size, Economic Development and Culture Change 6, 243-248.

Beckman, M. J. & Mcpherson, J. C., 1970, City size distribution in a central place hierarchy: An alternative approach, Journal of Regional Science 10, 25-33.

Beckman, M. J., 1974, Spatial equilibrium in the housing market, Journal of Urban Economics 1, 99-107.

Beesley, M. E., 1973, Urban Transport: Studies in Economic Policy,

Brueckner, J. K., 1980a, A vintage model of urban growth, Journal of Urban Economics 8, 389-402.

Brueckner, J. K., 1980b, Residential succession and land-use dynamics in a vintage model of urban housing, Regional Science and Urban Economics 10, 225-240.

Brueckner, J. K., 1981a, A dynamic model of housing production, Journal of Urban Economics 10, 1-14.

Brueckner, J. K., 1981b, Testing a vintage model of urban growth, Journal of Regional Science 21, 23-35.

Brueckner, J. K. & Burkhard von R., 1981, Dynamics of land-use for a closed city, Regional Science and Urban Economics 11, 1-17.

Brueckner, J. K., 1982, Building ages and urban growth, Regional Science and Urban Economics 12, 197-210.

Brueckner, J., 1987, The structure of urban equilibria: A unified treatment of the Muth-Mills model, in Handbook of Urban Economics (E. S. Mills, Ed) Vol. 2, 921-845, North-Holland, Amsterdam.

Castells, M., 1989, The Information city: Information technology, economic Restructuring, and the Urban-Regional Process, Basil Blackwell P157.

Curran, C. & Carlson, L. A., 1982, A theory of residential location decisions of two-worker households, Journal of Urban Economics 12, 102-114.

Dacey, M. F., 1965, The Geometry of Central Place Theory, Geografiska Annaler (Series B: Human Geography) 47, 111-124.

Dacey, M. F., 1966, Population of places in a central place hierarchy, "Journal of Regional Science 6, 27-33.

Fujita, M. & Ogawa, H., 1982, Multiple equilibria and structural transition of non-monocentric urban configurations, Regional Science and Urban Economics 12, 161-196.

Fujita, M., 1982, Spatial patterns of residential development, Journal of Urban Economics 12, 22-52.

Fujita, M., 1989, Urban economic theory: Land use and city size, Cambridge University Press.

Fujita, M. & Thisse, J. f., 1991, Spatial duopoly and residential structure, Journal of Urban Economics 30, 27-47.

Fujita, M., Thisse, J. f. & Zenou, Y., 1997, On the endogenous formation of secondary employment centers in a city, Journal of Urban Economics 41, 337-357.

Garreau J., 1991, Edge city: Life on the new frontier, Doubleday.

Gordon P. & Richardson H. W., 1996, Employment decentralization in US metropolitan areas: is Los Angeles an outlier or the norm, Environment and Planning A, 28, 1727-1743.

Henderson V and Mitra A., 1996, The new urban landscape: developers and edge cities, Regional Science and Urban Economics, 26, 613-643.

Henderson, V. & Slade, E., 1993, Development games in non-monocentric cities, Journal of Urban Economics 34, 207-230.

Jackson, K. T., 1985, Crabgrass Frontier: The Suburbanization of the United States, Oxford University Press.

Lind R. C., 1973, Spatial equilibrium, the theory of rents and the measurement of benefits from public programs, Quarterly Journal of economics, May 87, 188-207.

Mills, E. S. 1967, An aggregative model of resource allocation in a metropolitan area, American Economic Review 57, 197-210.

Mills, E. S. & Song, B-N, 1979, Urbanization and Urban Problems: Studies in the Modernization of The Republic of Korea: 1945-1075, Harvard University Press.

Mills, D. E., 1981, Growth, speculation and sprawl in a monocentric city, Journal of Urban Economics 10, 201-226.

Miyao, T., 1975, Dynamics and comparative statics in the theory of residential location, Journal of Economic Theory 11, 133-146.

Moses, L. N., 1962, Towards a theory of intra-urban wage differentials and their influence on travel patterns, *Papers Proc. Reg. Sci. Assoc.* IX, 53-63.

Ogawa, W. & Fujita, M., 1980, Equilibrium land use patterns in a nonmonocentric city, Journal of Regional Science 20, 455-475.

Parr, J. B., 1969, City hierarchies and the distribution of city size: A reconsideration of Beckman's contribution, Journal of Regional science 9, 239-253.

Pines D. & Sadka E., 1986, Comparative statics analysis of a fully closed city, Journal of Urban Economics, 20, 1-20.

Pine D. & Weiss Y., 1976, Land improvement projects and land values, Journal of Urban Economics, Jan 3 1-13.

Ross, S., & Yinger, J., 1995, Comparative static analysis of open urban models with a full labor market and suburban employment, Regional Science and urban Economics 25, 575-605.

Rothenberg J., 1965 "Urban Renewal Programs," in Robert Dorfman, ed., Measuring the Benefits of Government Investment, Wahdington

Sasaki, K., 1987a, A Comparative static analysis of urban structure in the setting of endogenous income, Journal of Urban Economics, 22, 53-72.

Sasaki, K., 1987b, The effects of urban transportation system change on industrial and residential areas, Discussion paper No.21 (Research Center for Applied Information Sciences, Tohoku University, Sendai).

Sasaki, K., 1989, Transportation system change and urban structure in Two-transport Mode Setting, Journal of Urban Economics, 25, 346-367.

Sasaki, K., 1990, Income class, modal choice, and urban spatial structure, Journal of Urban Economics, 27, 322-343.

Sasaki, K., 1990, The establishment of a subcenter and urban spatial structure, Environment and Planning A 22, 369-383.

Sasaki, K., and M. Kaiyama, 1990, The effects of urban transportation costs on urban spatial structure with endogenous wage Income, Regional Science and Urban Economics 20, 223-244.

Sasaki, K., and Mun, S., 1996, A Dynamic Analysis of Multiple-Center Formation in a City, Journal of Urban Economics 40, 257-278.

Solow R. M. & Vickrey W. S., 1971, Land use in a long narrow city, Journal of Economic

Theory, 3, 430-447.

Solow R. M., 1973, Congestion cost and the use of land for streets, The Bell Journal of Economics and Management Science, 4, 601-618.

Solow, R., 1973, On equilibrium models of urban spatial structure, in: J. Perkins, ed., Essays in modern economics. London: Longmans.

Straszheim, M. R., 1984, Urban agglomeration effects and employment and wage gradients, Journal of Urban Economics 16, 187-207.

Sullivan A. M., 1983 A general equilibrium model with external scale economies in production, Journal of Urban economics 13, 235-255.

Sullivan, A.M., 1986, A general equilibrium model with agglomerative economies and decentralization employment, Journal of Urban Economics, 20, 55-74.

Toffler A., 1980, The Third Wave, New York, William Morrow.

Wheaton, W., 1974, A Comparative static analysis of urban spatial structure, Journal of Economic Theory, 9, 223-237.

Wheaton, W., 1977, Residential decentralization, land rents, and the benefits of urban transportation investment, Vol. 67, Num2, 138-143.

Wheaton W., 1979 Monocentric models of urban land use: Contributions and criticisms, in: P. Mieszkowski and M. Strszheim, eds, Current issues in urban economics (Johns Hopkins University Press, Baltimore. MD)

Wheaton, W. C., 1982, Urban residential growth under perfect foresight, Journal of Urban Economics 12, 1-21.

Wheaton, W. C., 1982, Urban spatial development with durable but replaceable Capital, Journal of Urban Economics 12, 53-67.

White, M.J., 1976, Firm suburbanization and urban subcenters, Journal of Urban Economics 3, 323-343.

White, M., 1977, A model of residential location choice and commuting by men and women workers, Journal of Research Science 1, 41-52.

White, M., 1988, Location choice and commuting behavior in cities with decentralized employment, Journal of Urban Economics 24, 129-152.

Wieand K. F., 1987, An extension of the monocentric urban spatial equilibrium model to a multicenter setting: The case of the two-center city, Journal of Urban Economics 21, 259-271.

Witchard L., 1984, A comparative static analysis of the optimum town, Journal of Urban Economics 15, 259-169.

Yinger, J., 1992, City and suburb: urban model with more than one employment center, Journal of Urban Economics 31, 181-205.

Zhang, Y., and K. Sasaki, 1997, Effects of subcenter formation on urban spatial structure, Regional Science & Urban Economics.

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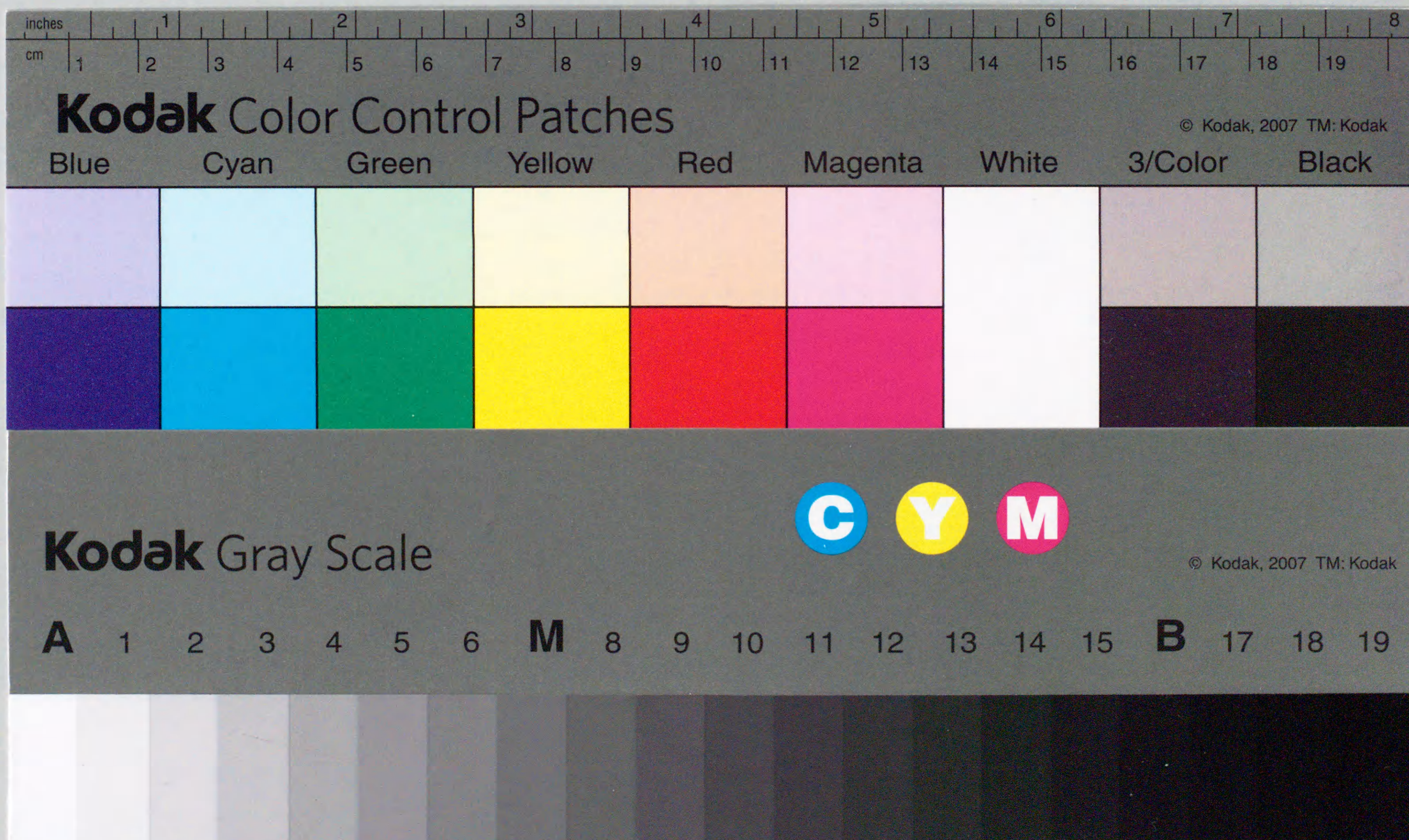
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